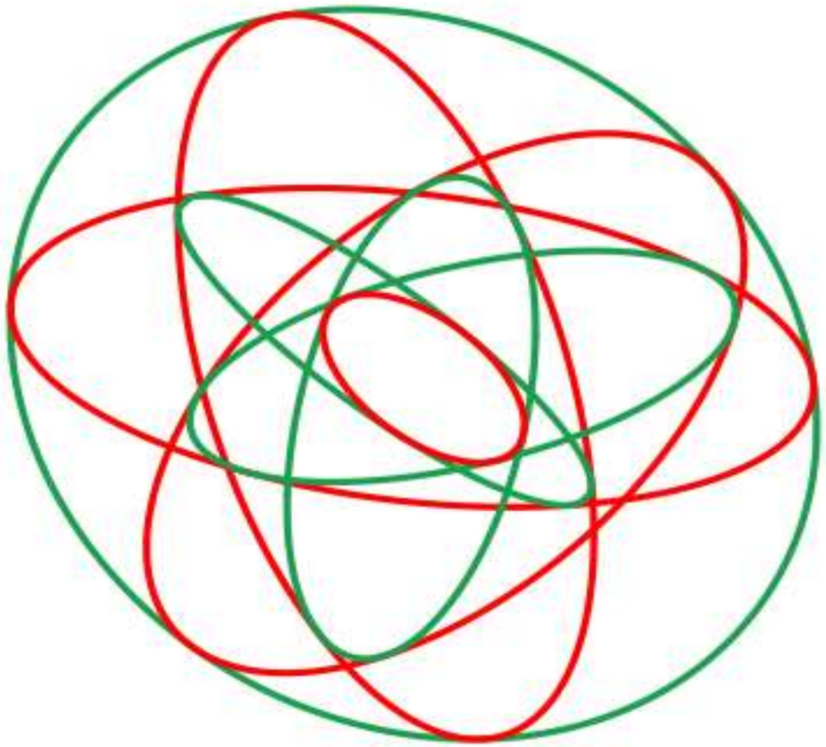
Die Wiederentdeckungen eines

geometrischen Satzes

von und mit

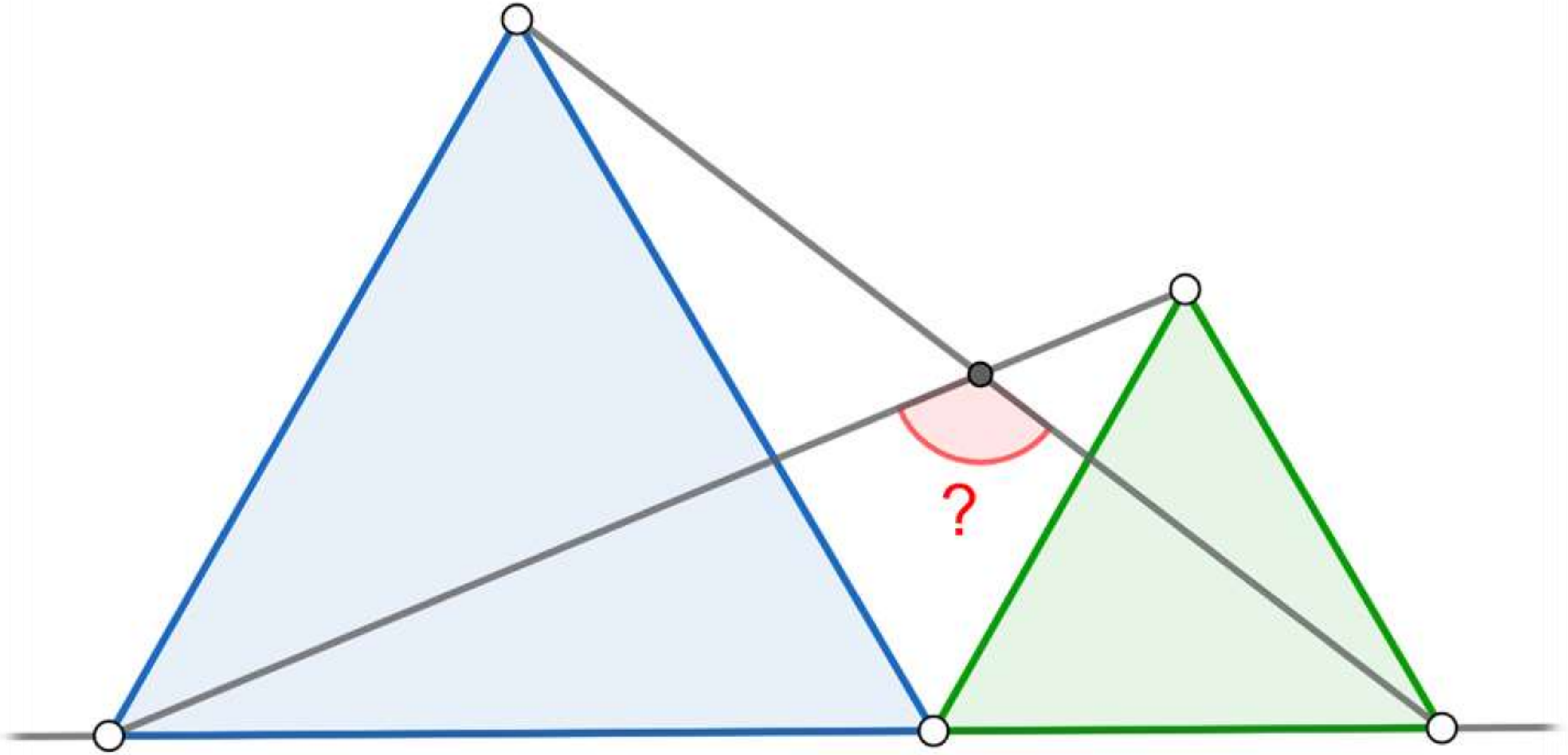
Roger Penrose



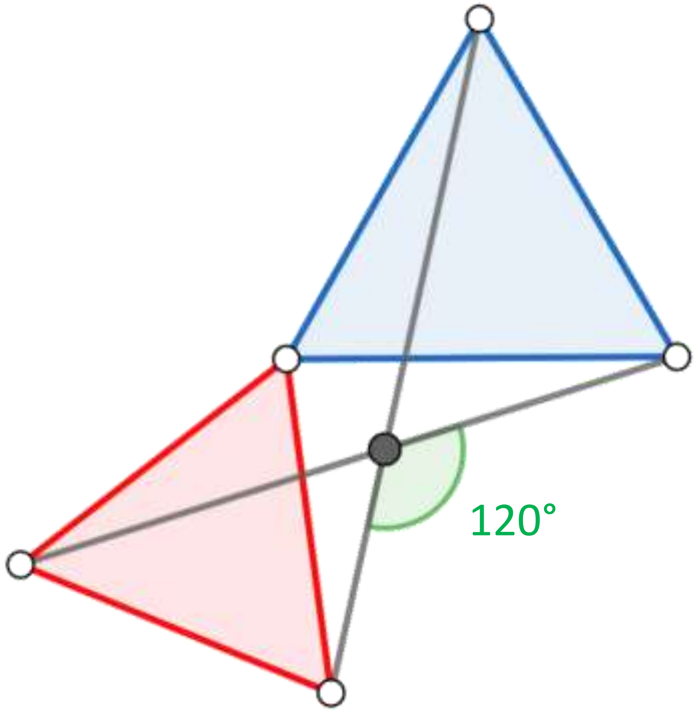
Thomas Neukirchner, FWS Karlsruhe

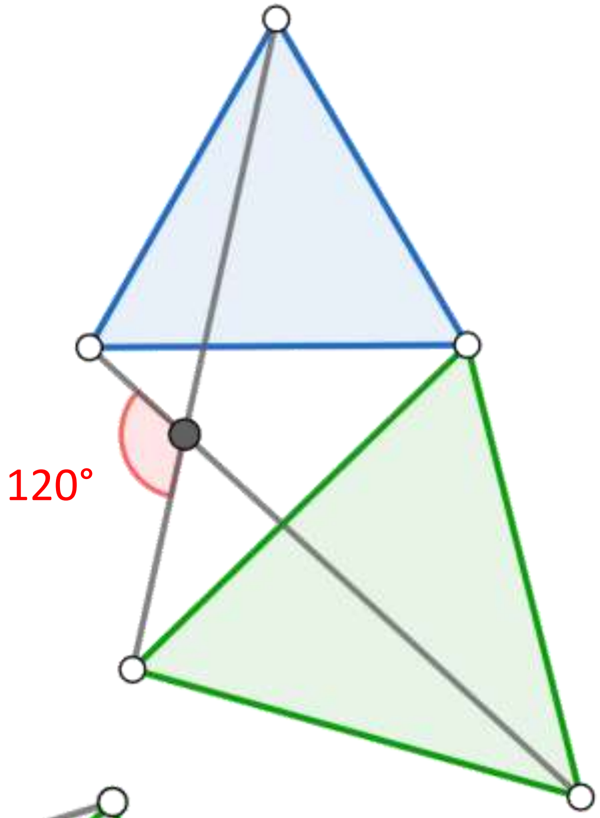
t.neukirchner@fws-ka.de

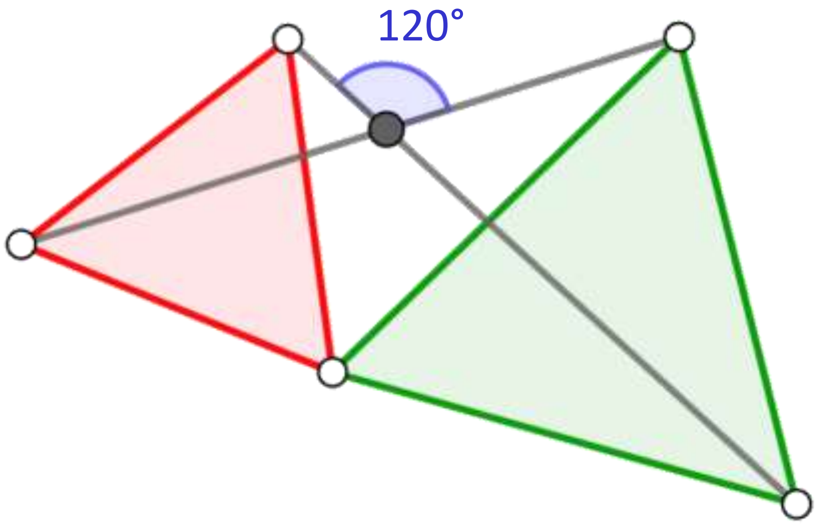
Mathematiklehrertagung Berlin, 17.-19. Januar $(20+25)^2$

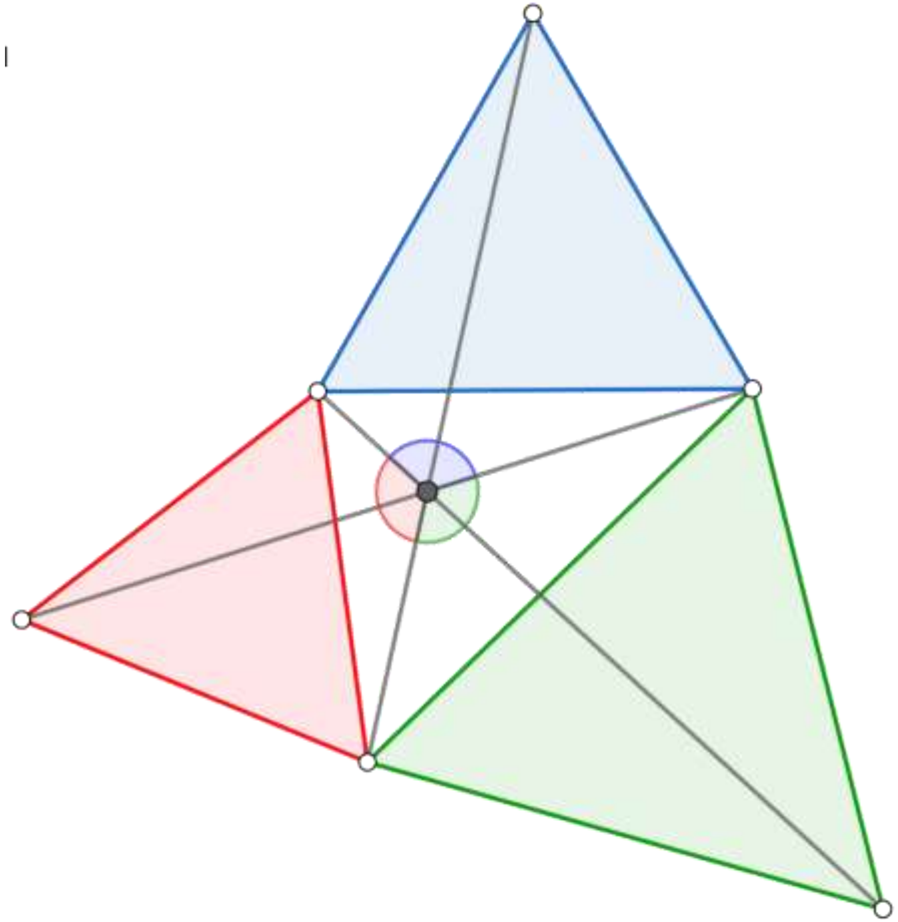


Aus: *Geometry Snacks* von Ed Southall and Vincernt Pantaloni



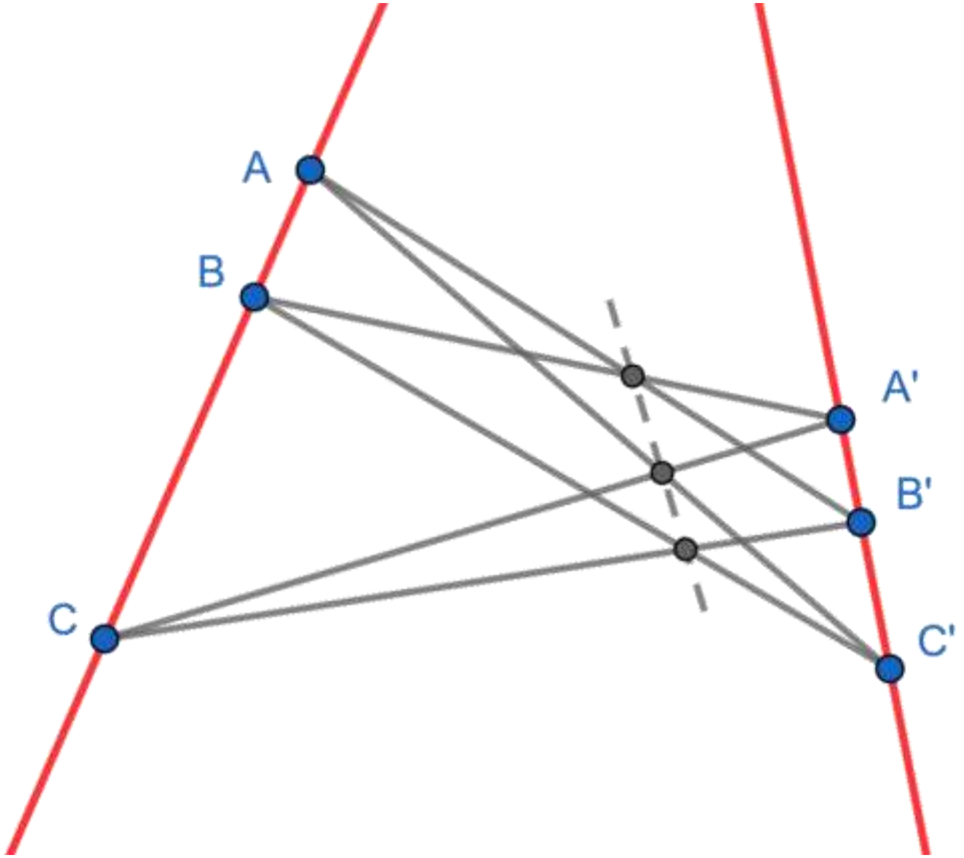




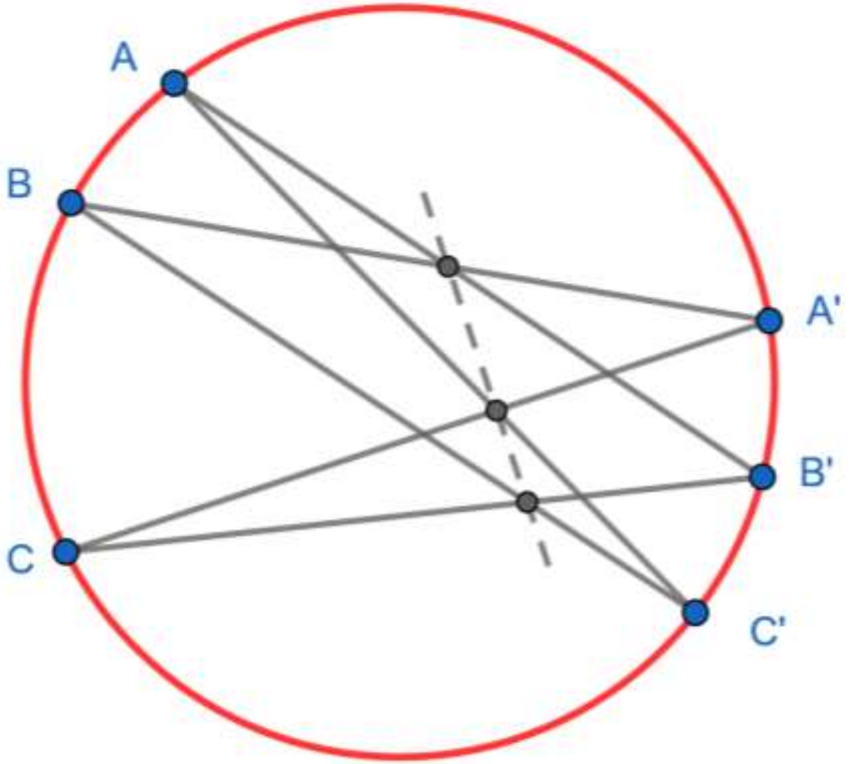


Warum treffen sich die 3 Geraden in einem Punkt ?

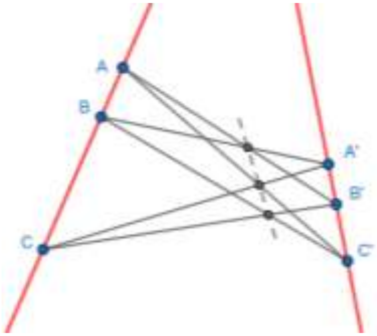
Satz von Pappos (340)

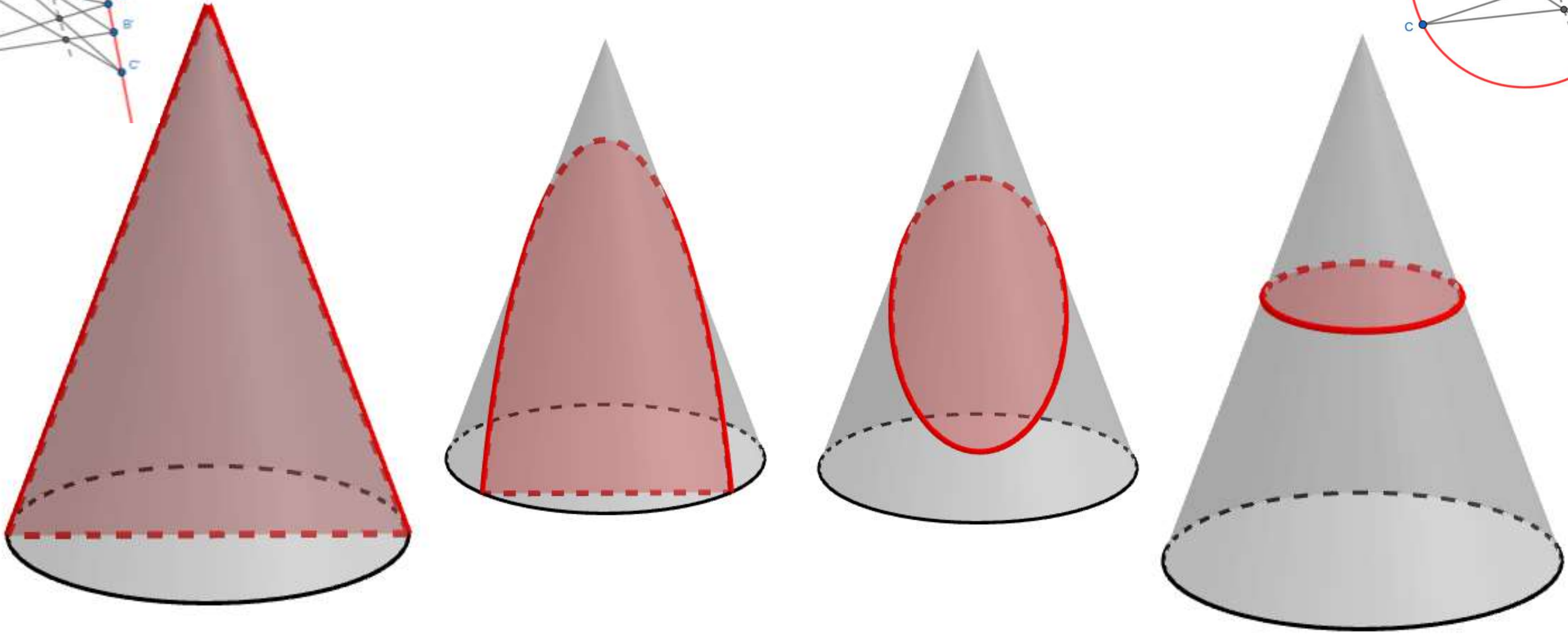


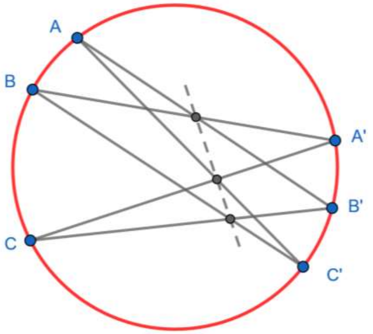
Satz von Pascal (1640)



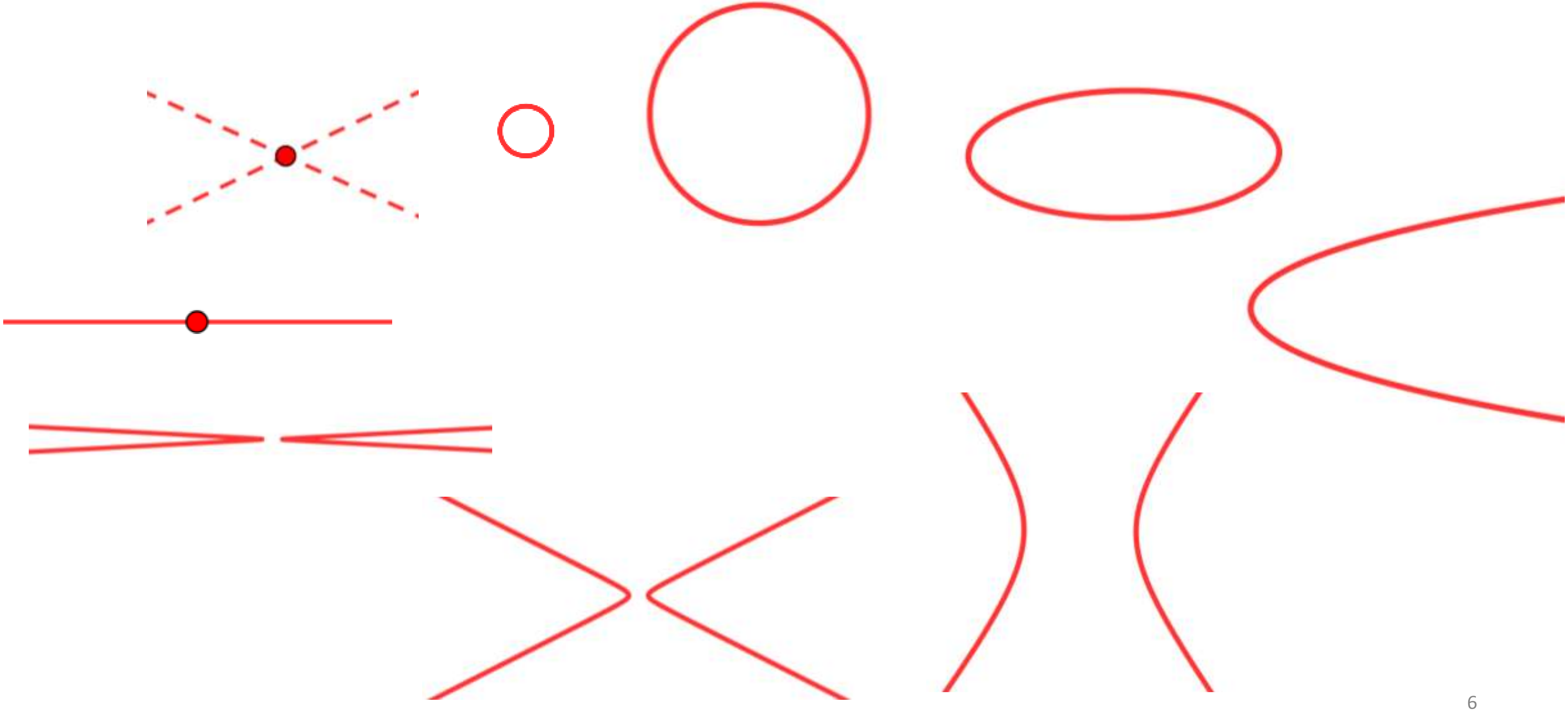
Verwandlungen I



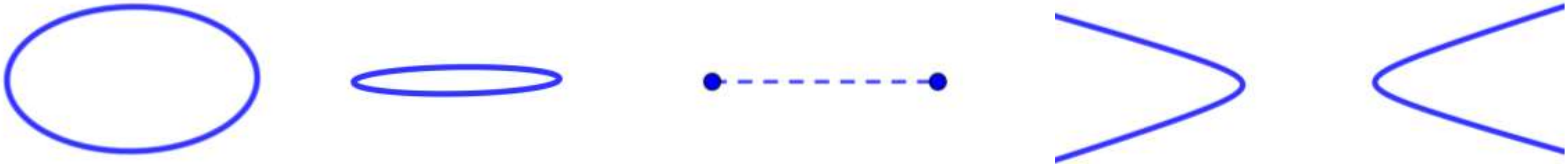




Verwandlungen I

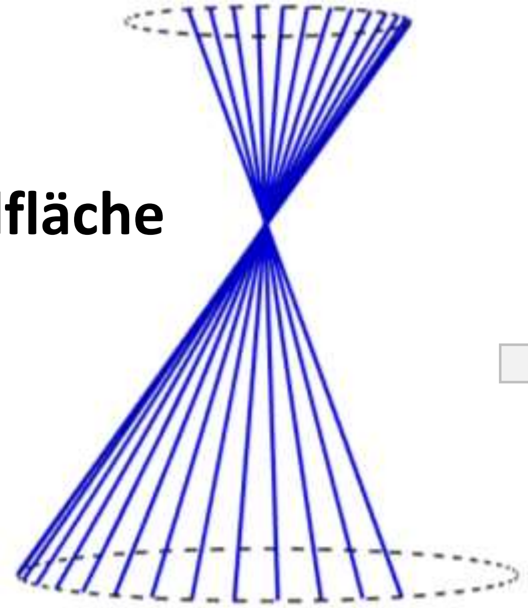


Verwandlungen I

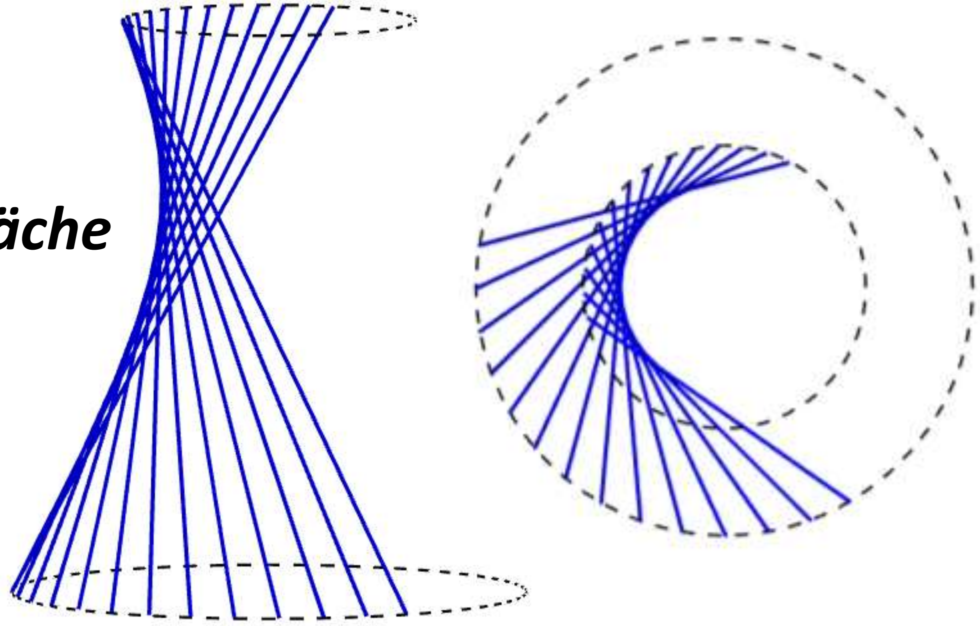


Ein Punktepaar ergibt sich *nicht* als Kegel**schnitt.** Es hat eher etwas mit **Sicht** zu tun...

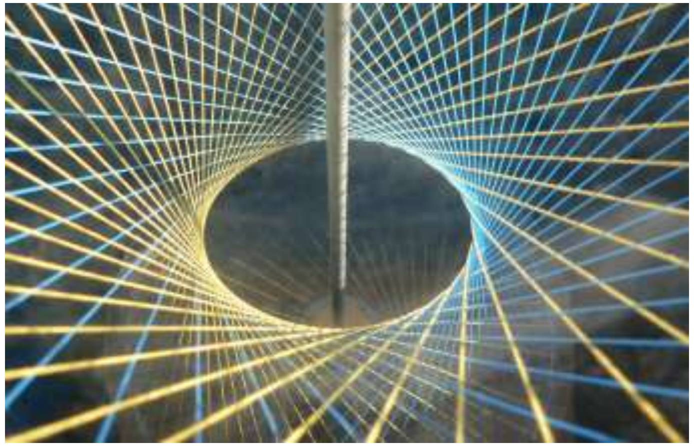
Kegelfläche

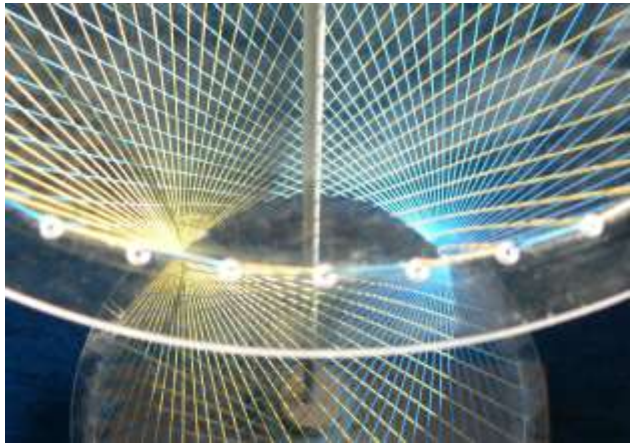


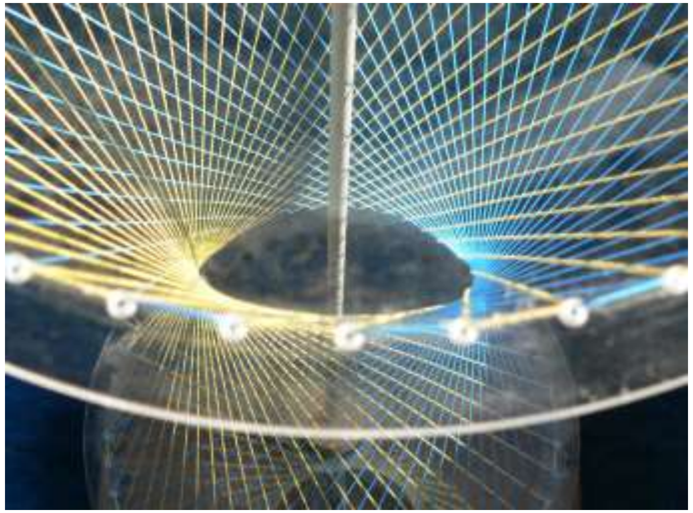
Regelfläche

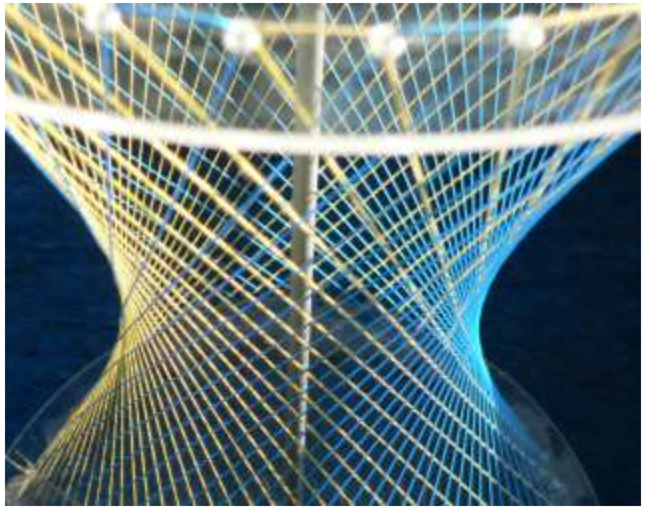


Verwandlungen I

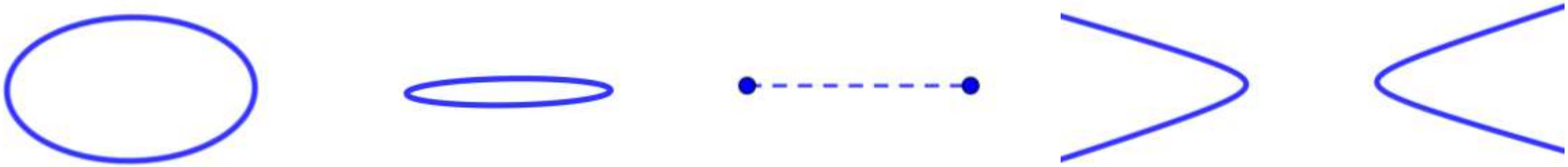


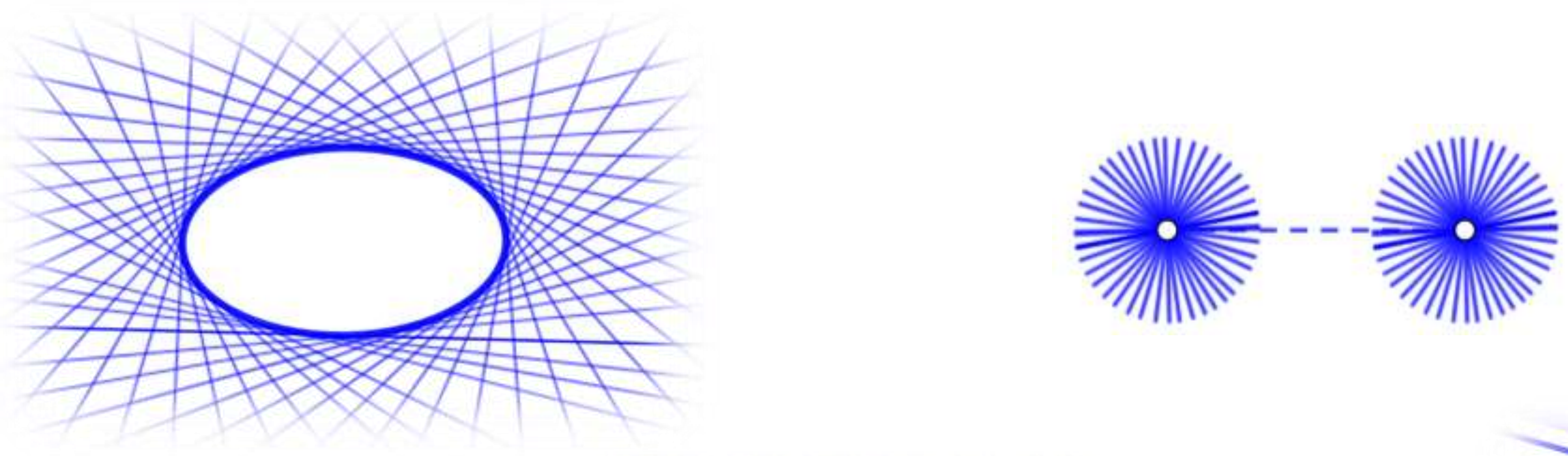


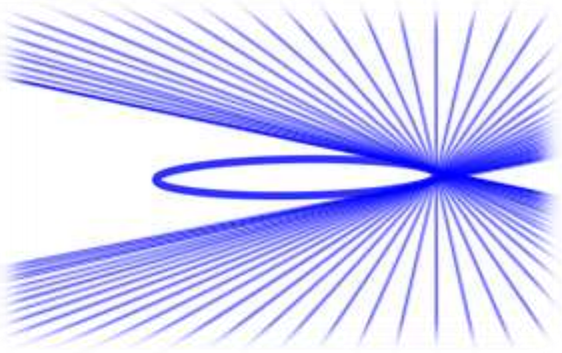


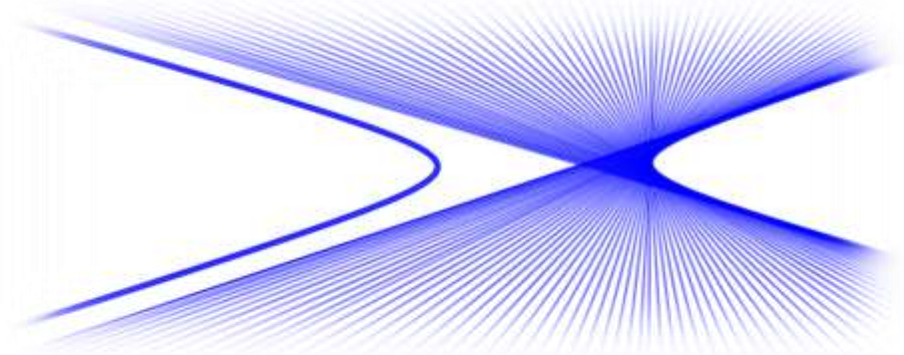


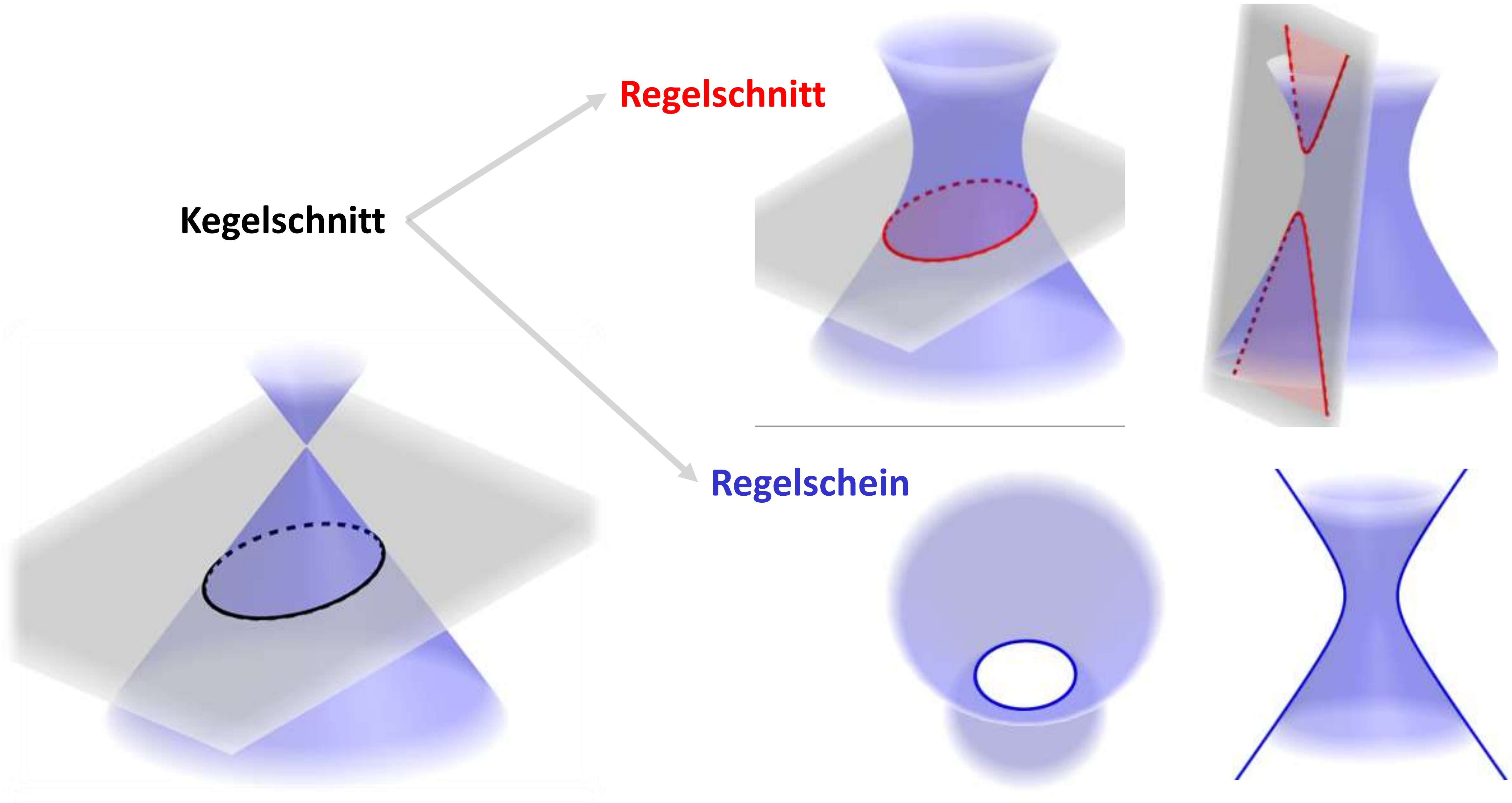
Verwandlungen I





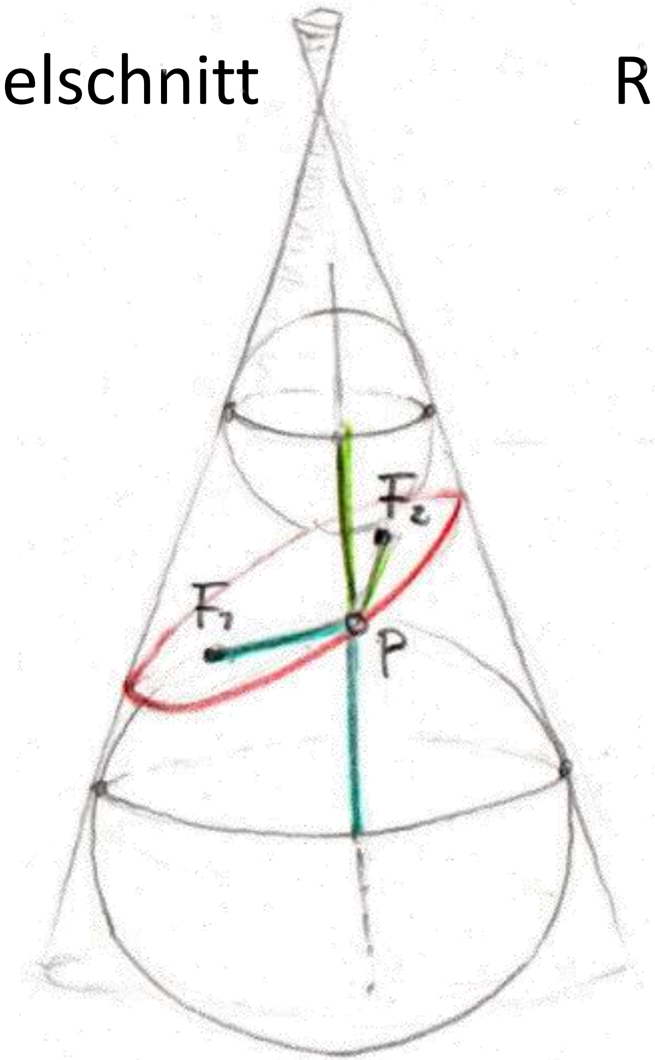




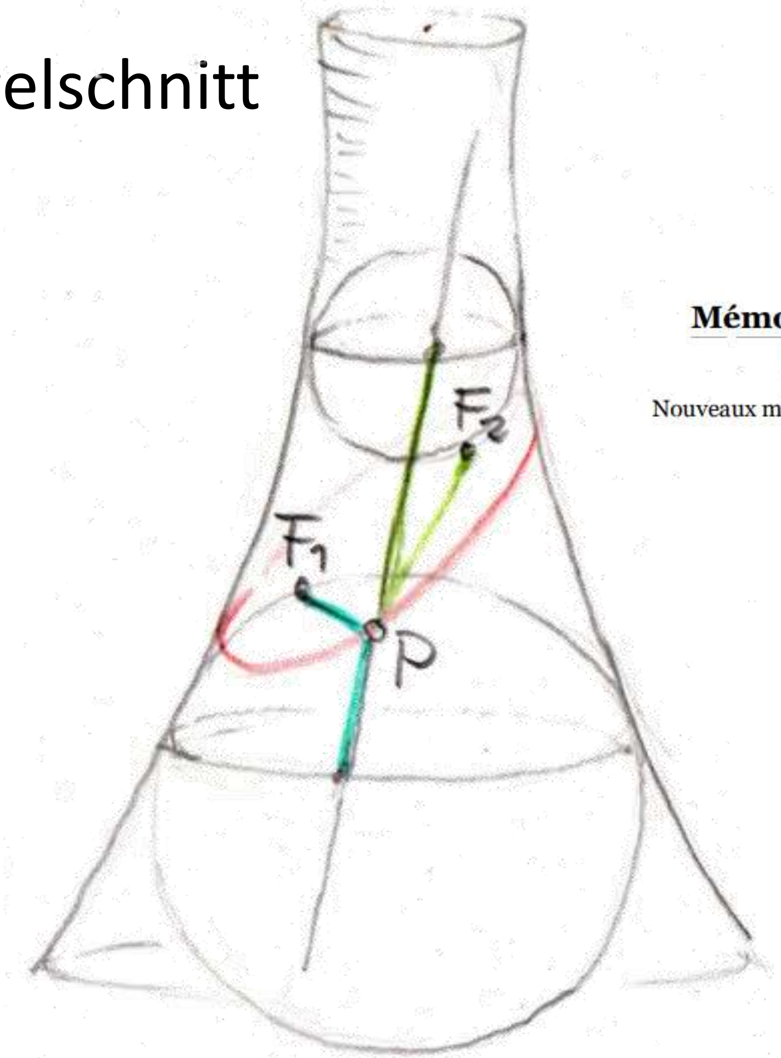

Kegelschnitt
Regelschnitt
Regelschein

Dandelin‘schen Kugeln (1826)

Kegelschnitt



Regelschnitt



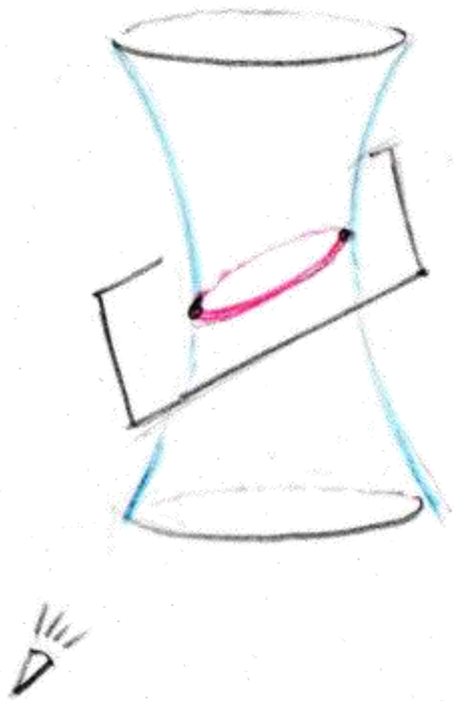
Germinal Pierre Dandelin

Mémoire sur l’hyperboloïde de révolution, et sur les hexagones de Pascal et de M. Brianchon

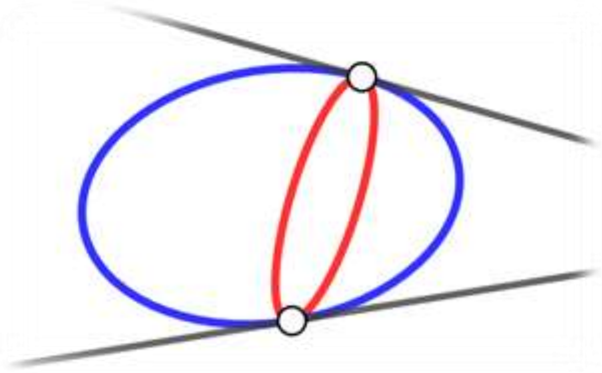
Nouveaux mémoires de l’Académie Royale des Sciences et Belles-Lettres de Bruxelles, T. III., 1826 (pp. 3-16).

Gleichzeitig Schnitt & Schein

Regelschein & Schein eines Regelschnitts



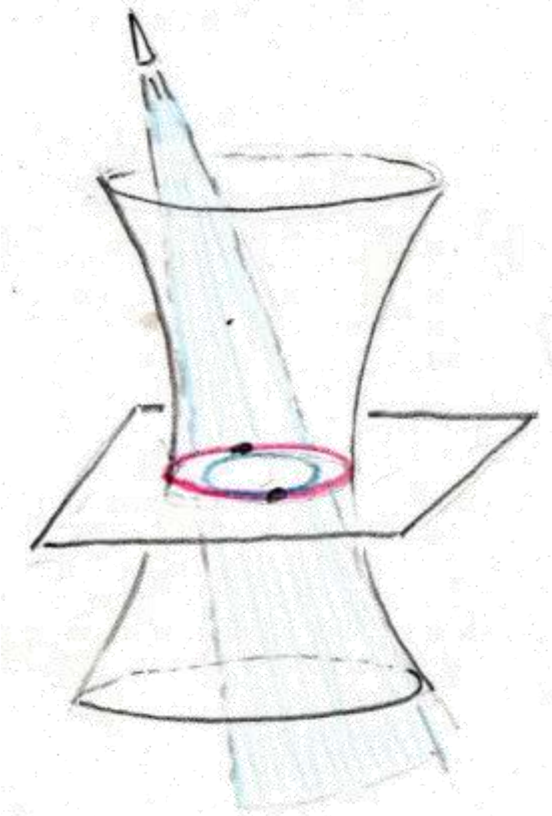
Doppelkontakt



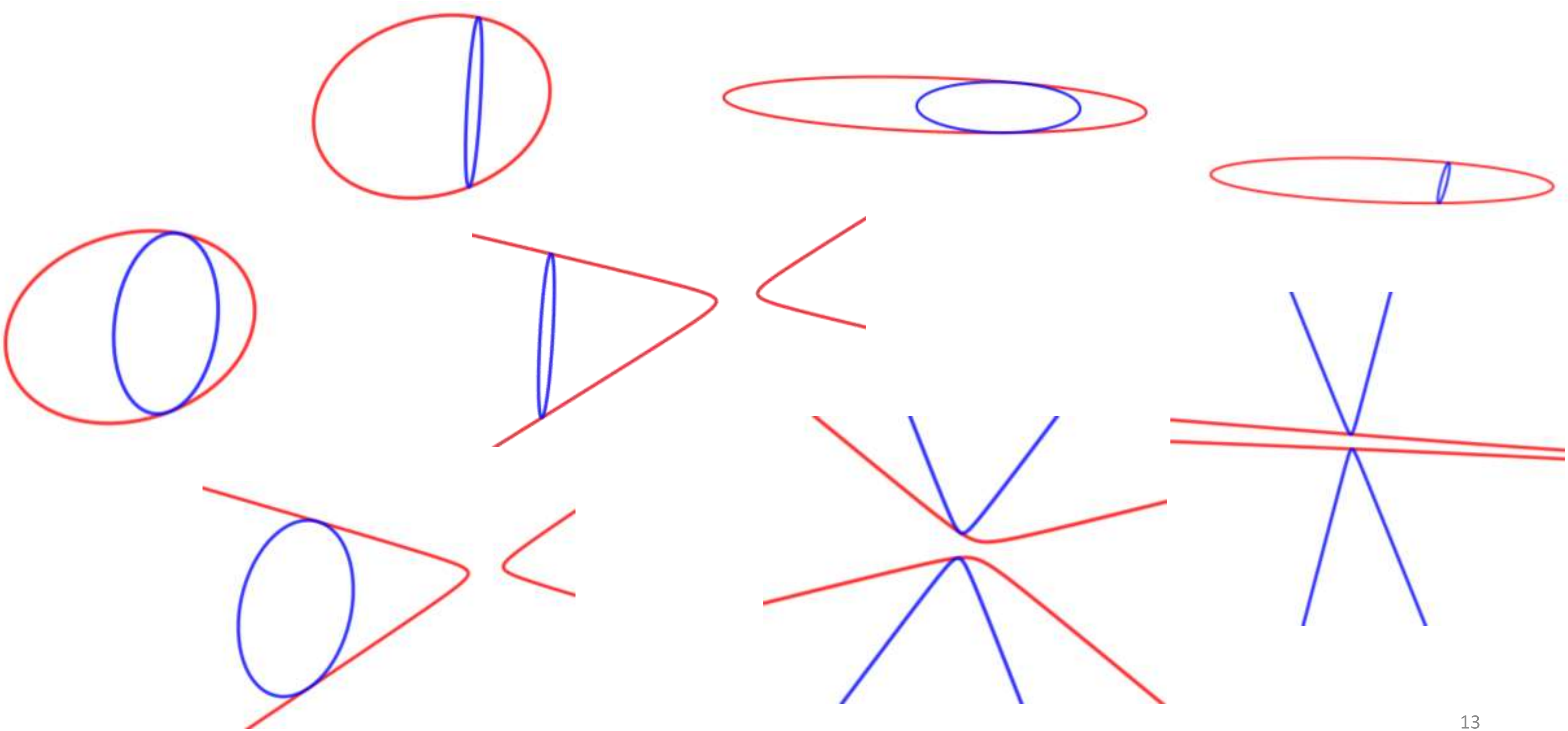
Paare von Regelschnitten mit:

- 2 gemeinsame Punkte
- 2 gemeinsame Tangenten

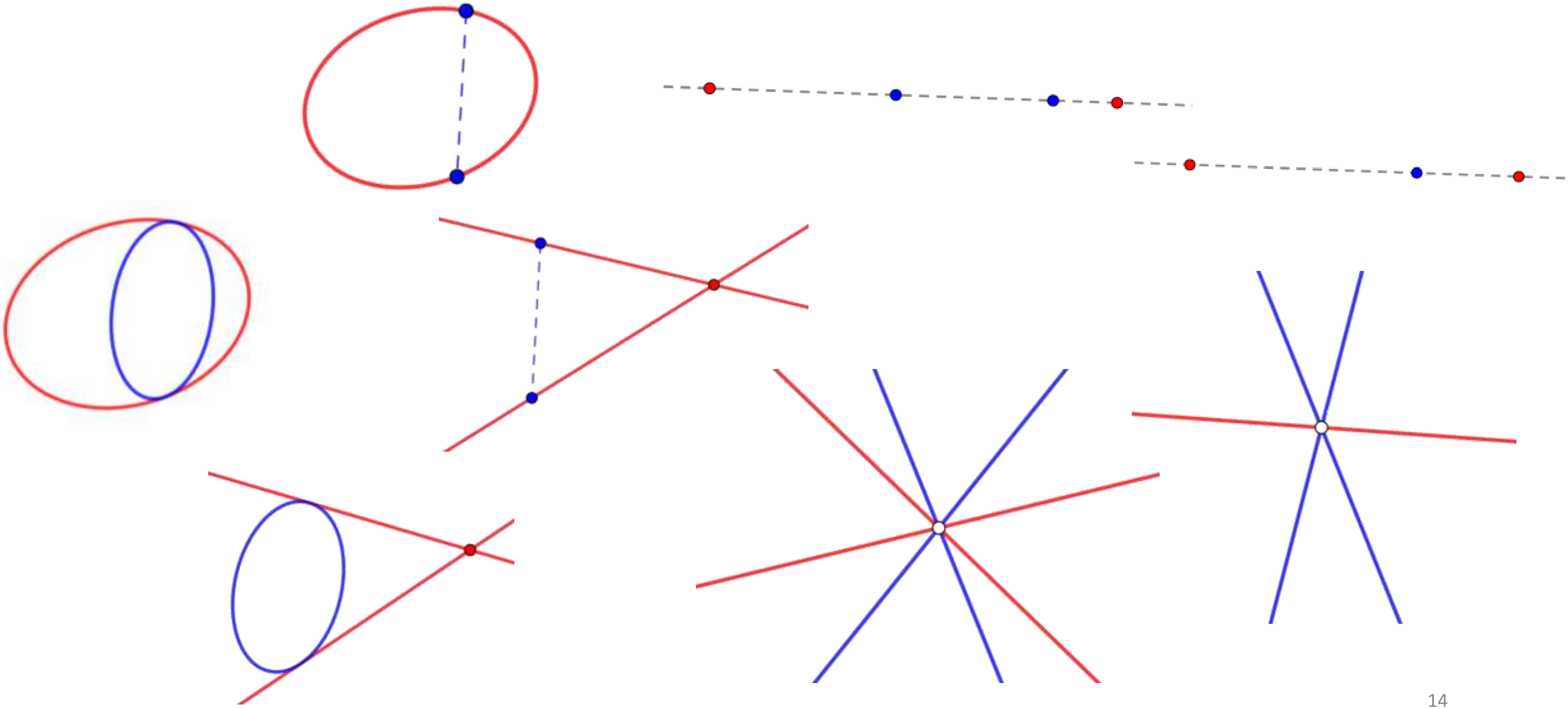
Regelschnitt & Schnitt eines Regelscheins:



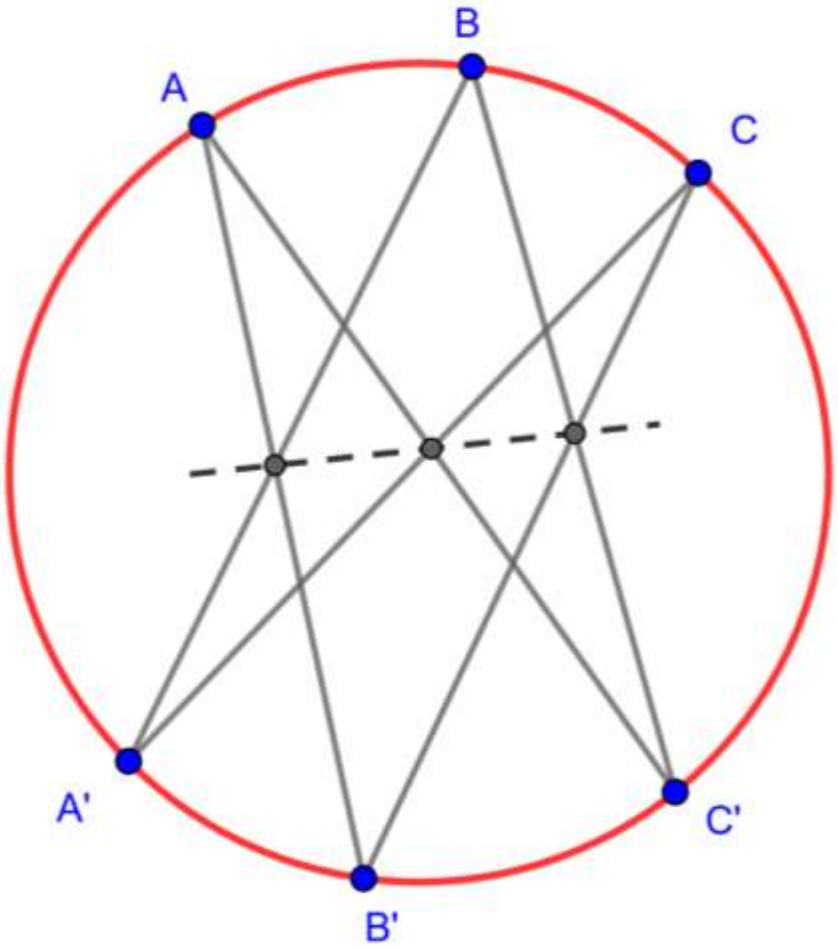
Verwandlungen II: Regelschnitte im Doppelkontakt

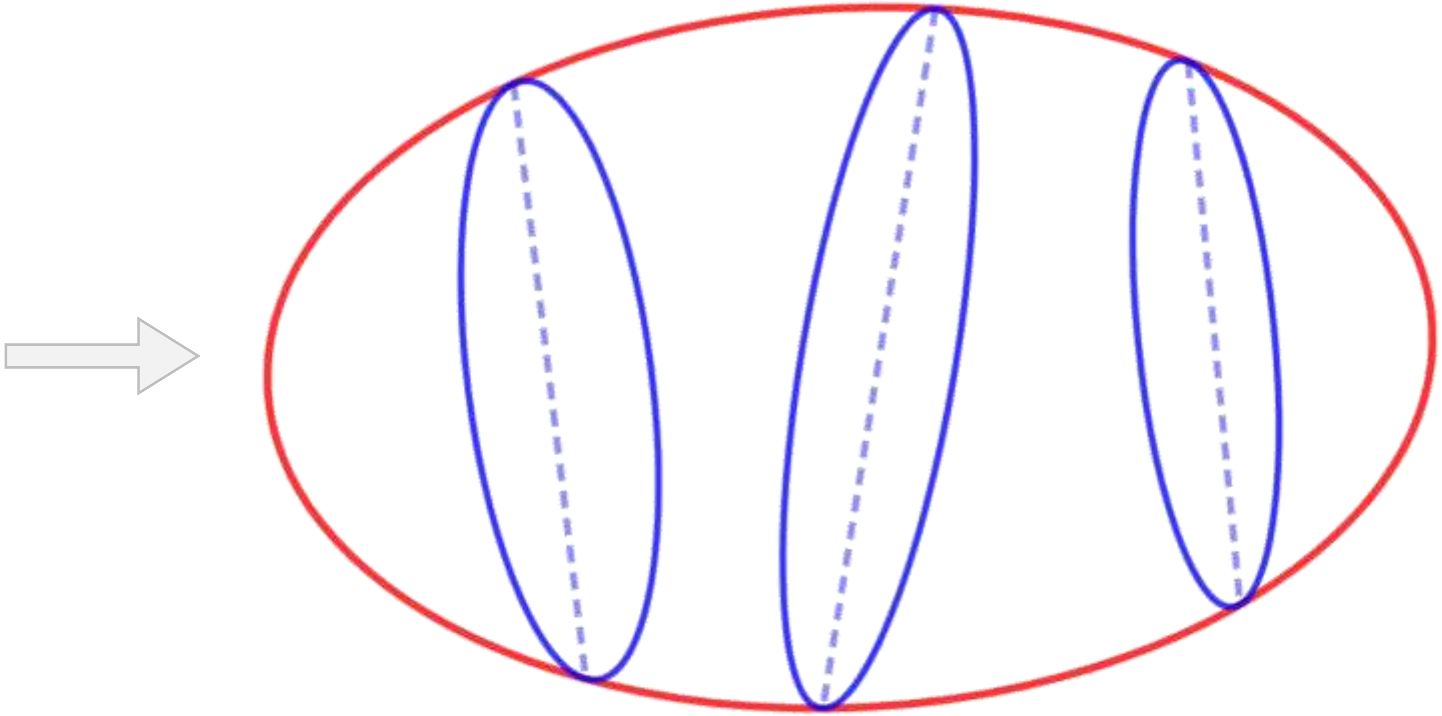


Verwandlungen II: Grenzfälle

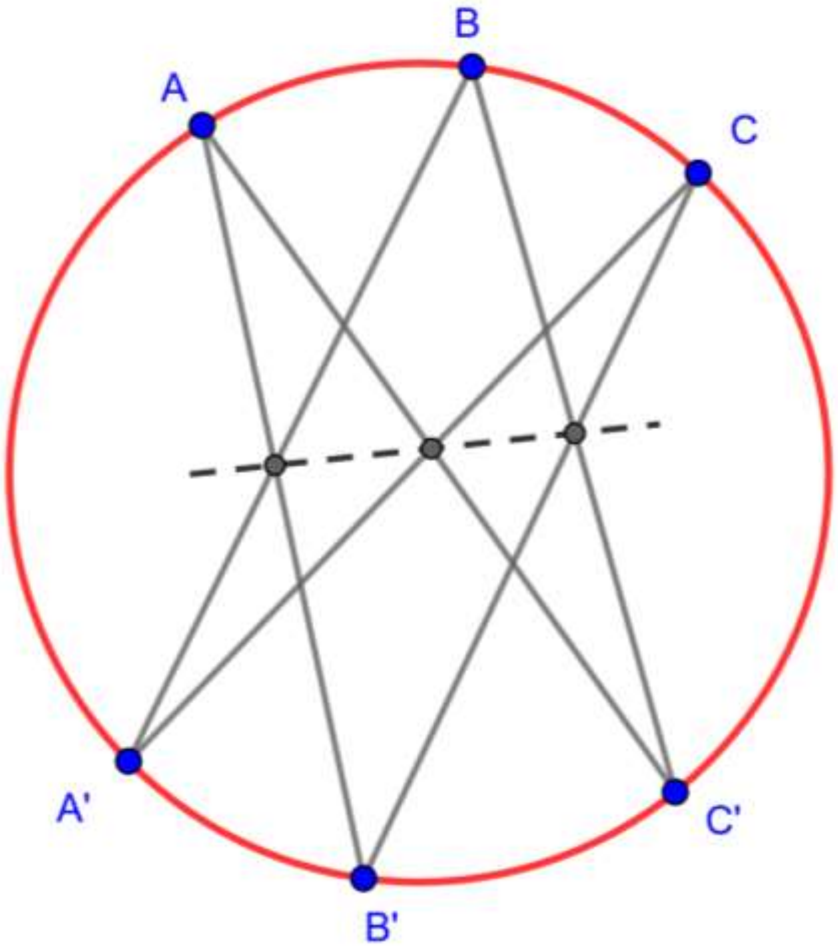


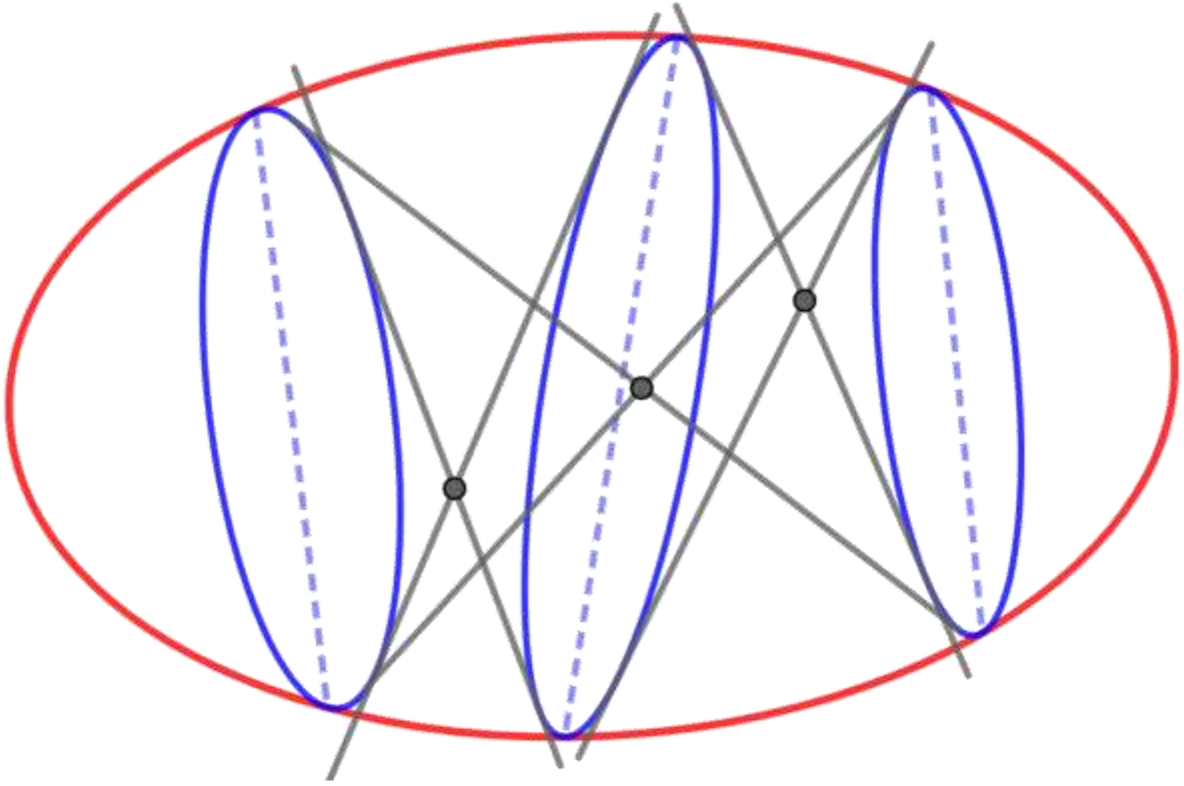
Verwandlung von Pascal (1640) zu Salmon (1855)



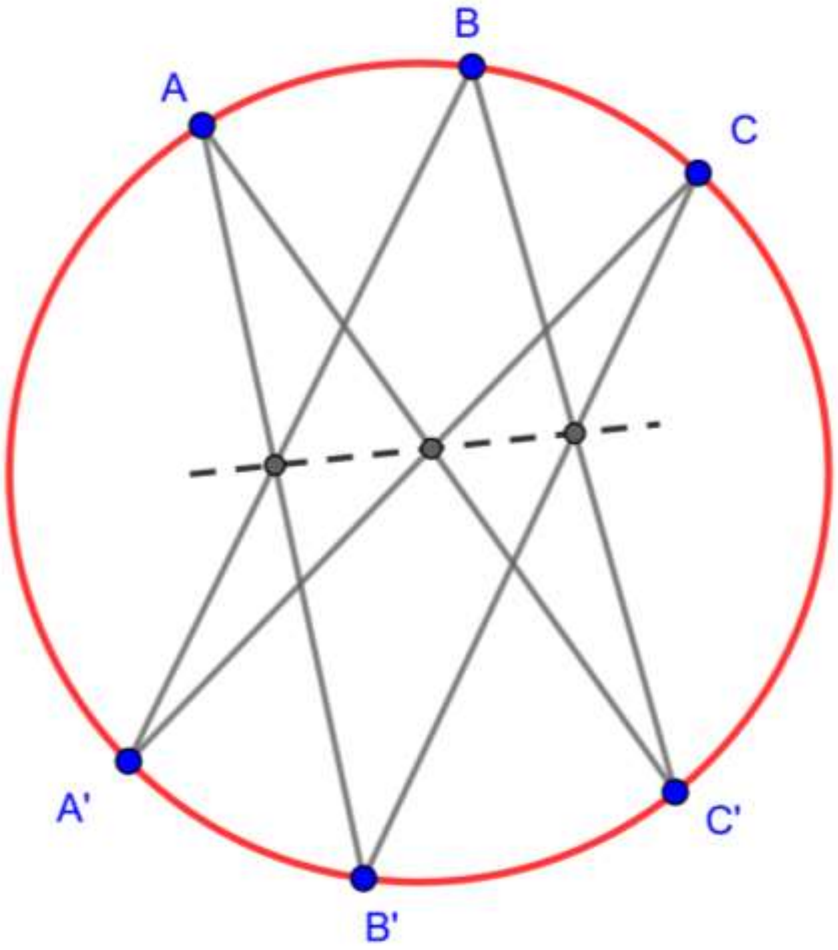


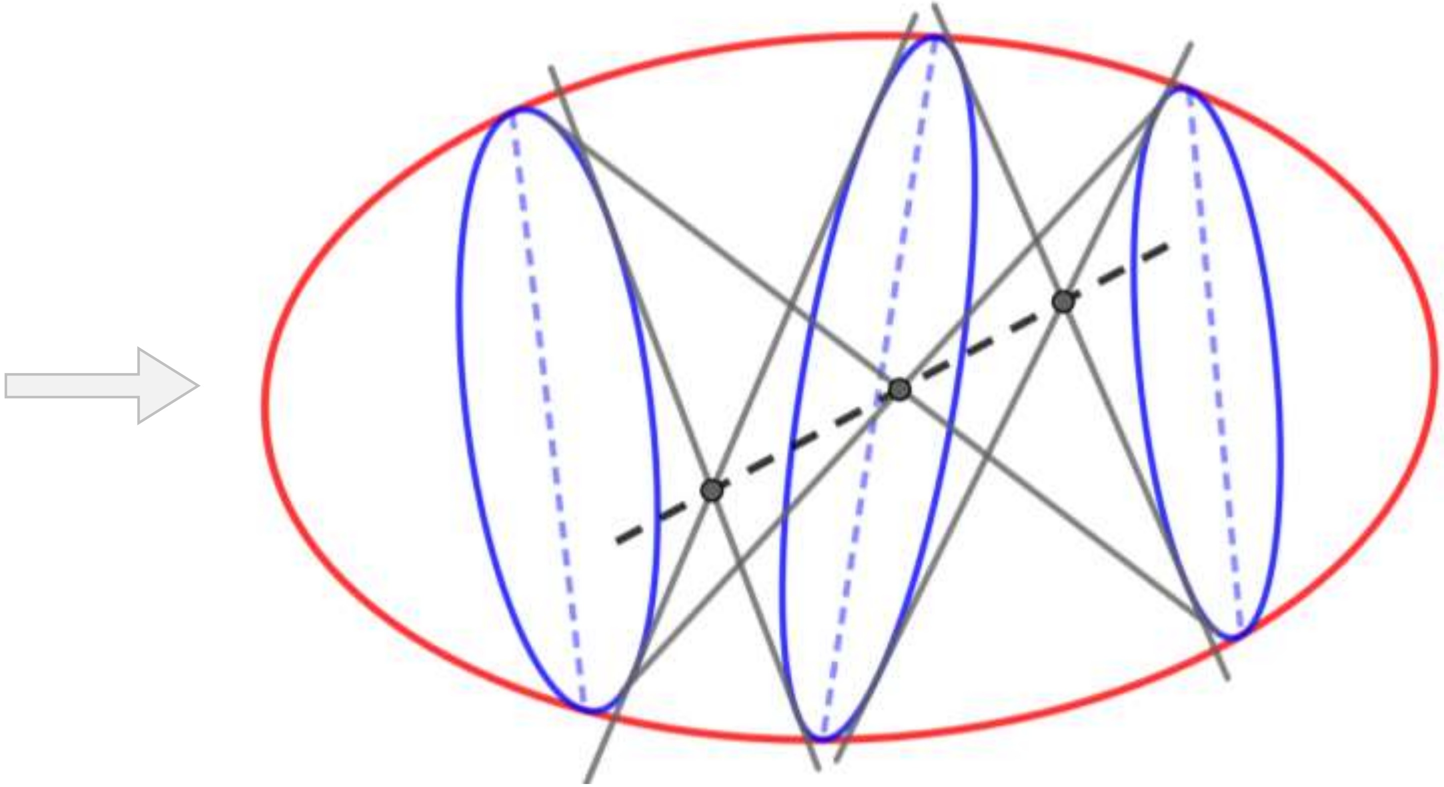
Verwandlung von Pascal (1640) zu Salmon (1855)



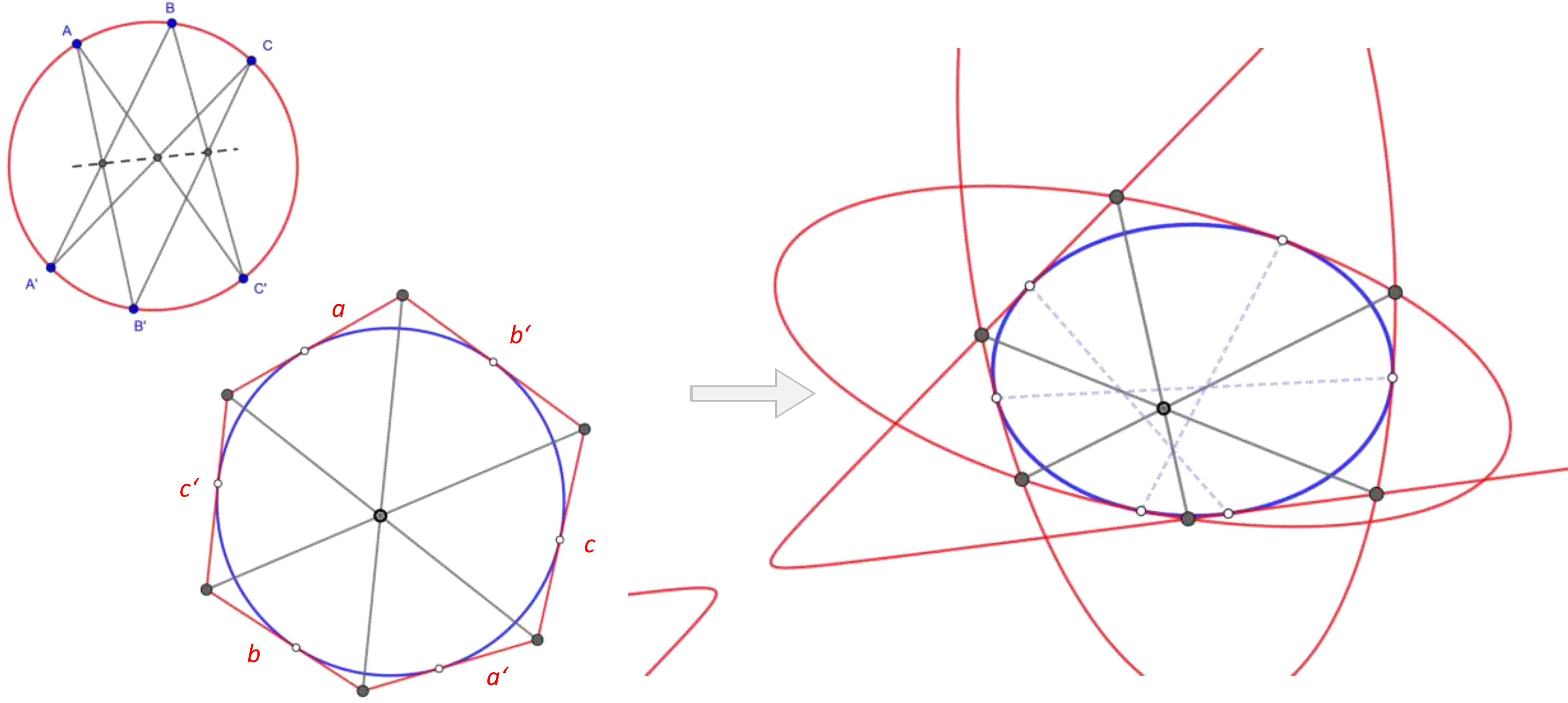


Verwandlung von Pascal (1640) zu Salmon (1855)

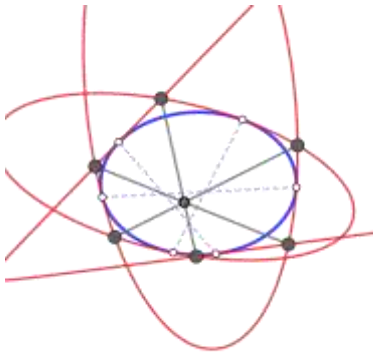


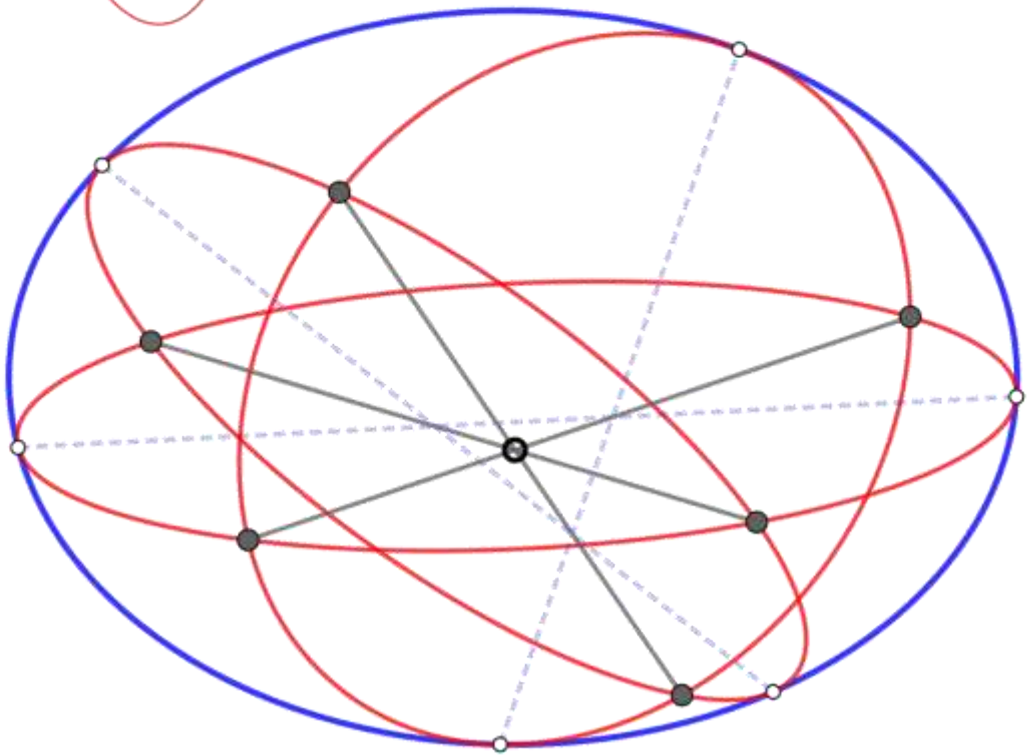


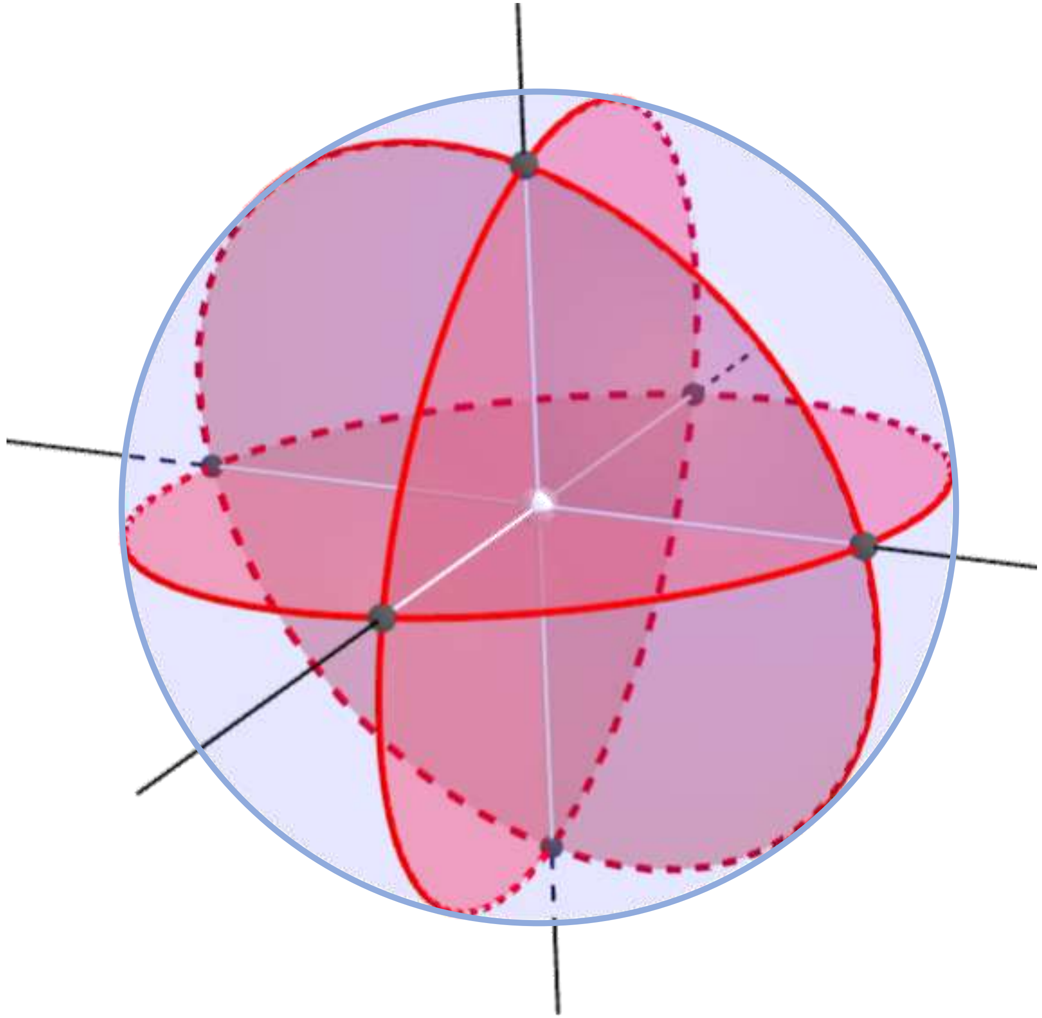
Verwandlung von Brianchon (1806) zu Salmon (1855)



Salmon als räumliche Konfiguration



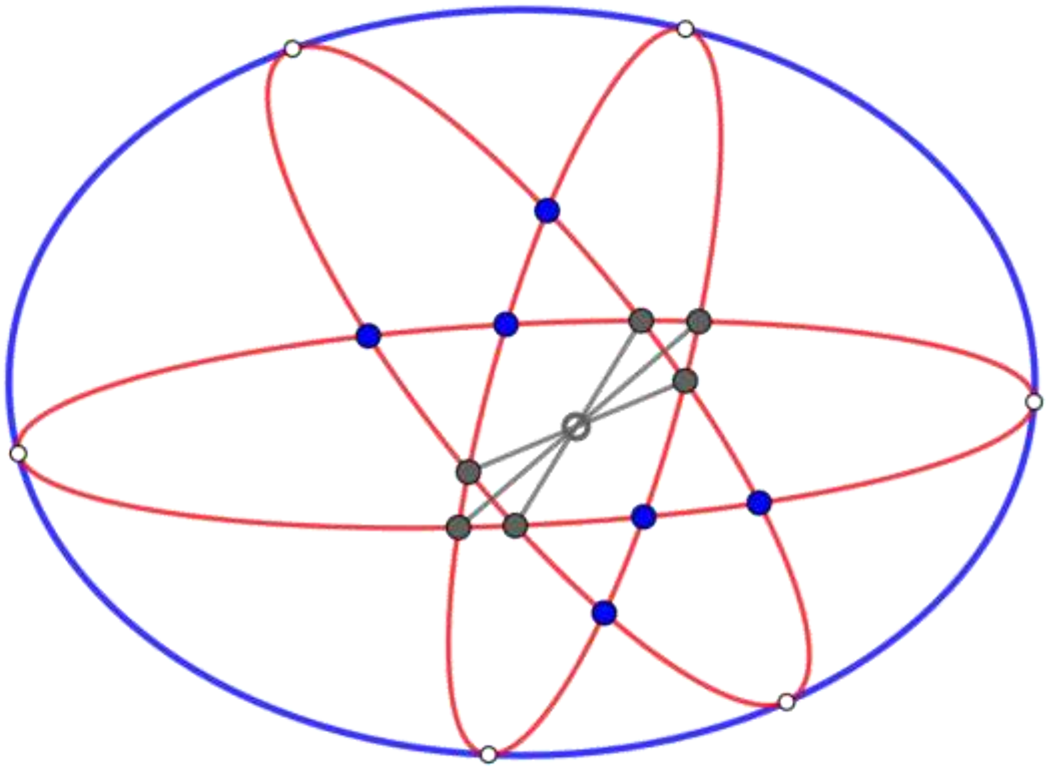


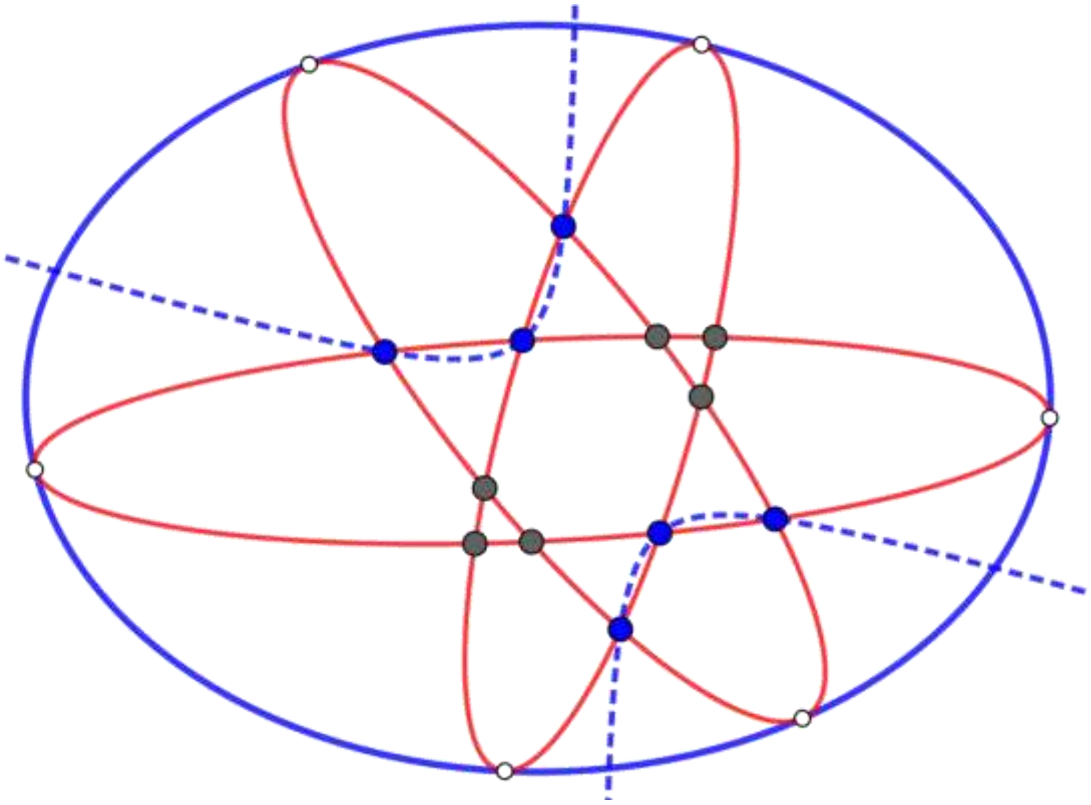


Schein **und** **3 Schnitte** **einer Sphäre**

(Sphäre = Regelfläche mit imaginären Geraden)

Von Salmon (1855) zu Evelyn (1974)

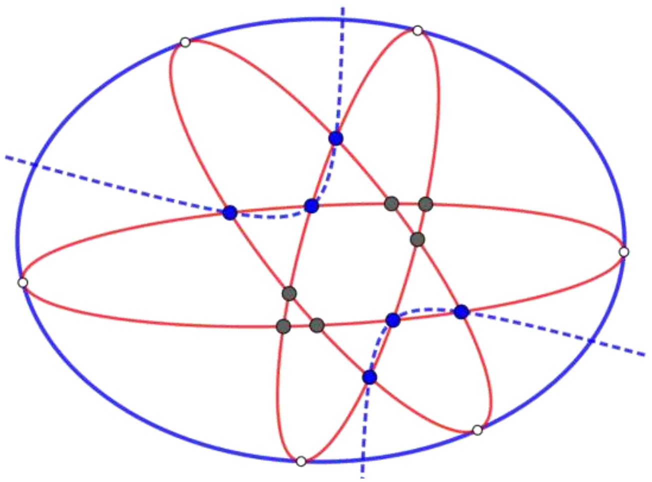


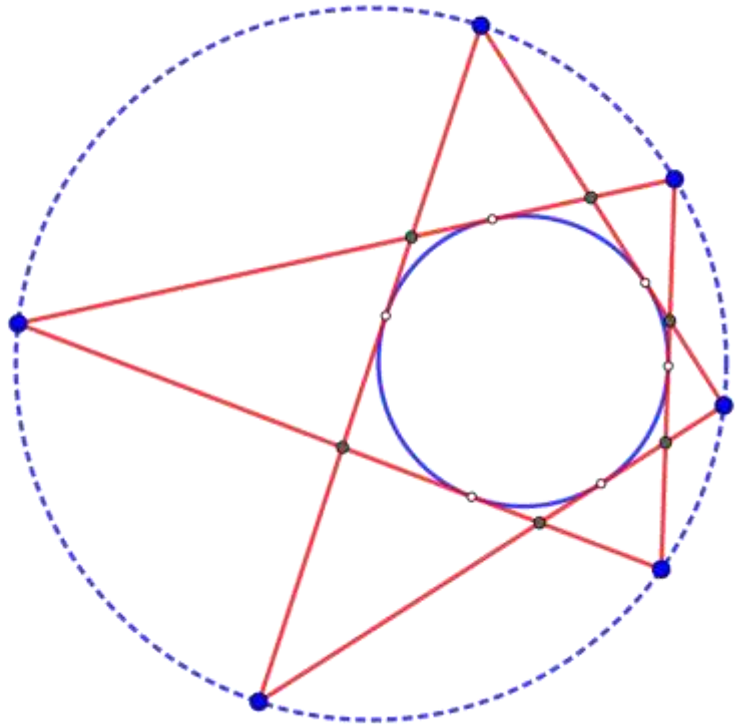


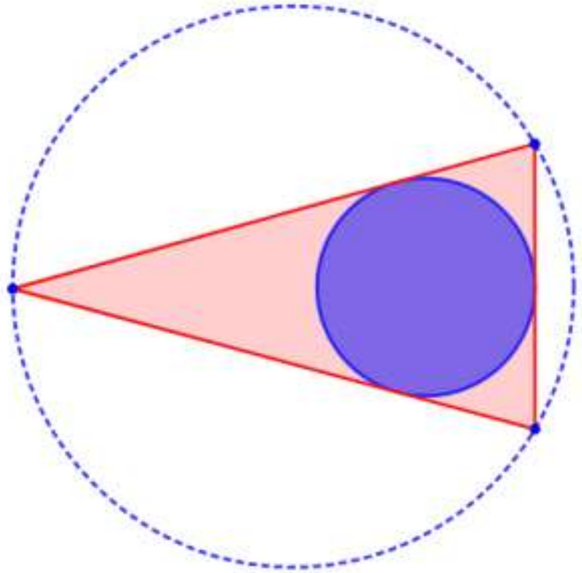
The Seven Circles Theorem
and other new theorems

C. J. A. Evelyn
G. B. Money-Coutts
J. A. Tyrrell

Von Evelyn (1974) zu Poncelet für Dreiecke (1822)







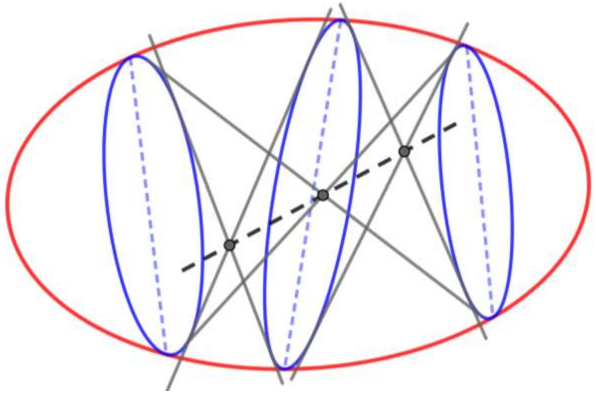
Poncelet: Ist ein Dreieck einem Kegelschnitt umschrieben und einem anderen Kegelschnitt einbeschrieben, dann gibt es unendlich viele solcher Dreiecke.

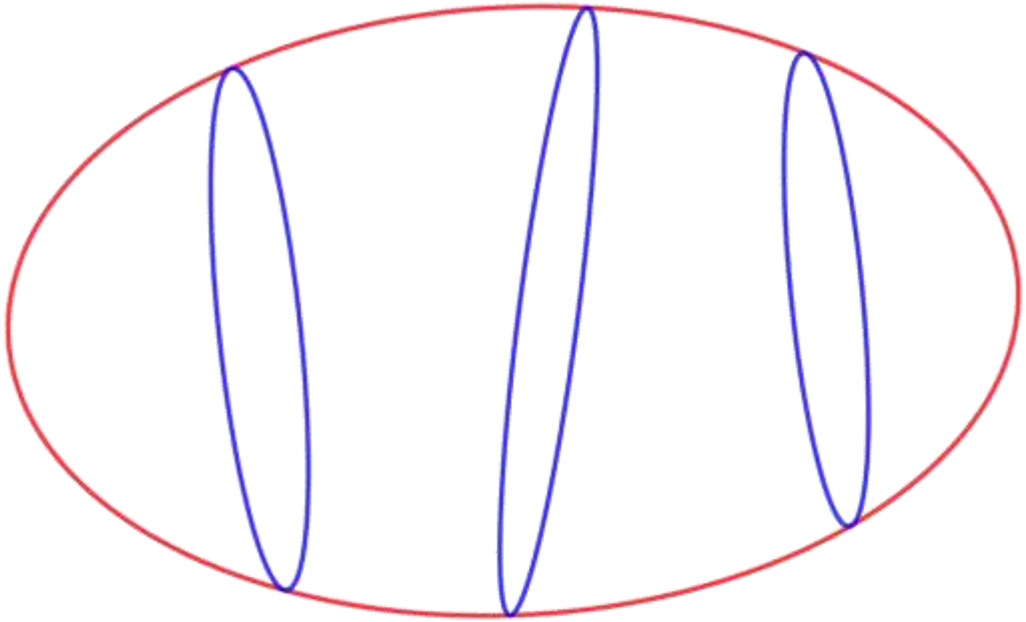
Frage: *Kann man Poncelet aus dem Räumlichen heraus verstehen?*

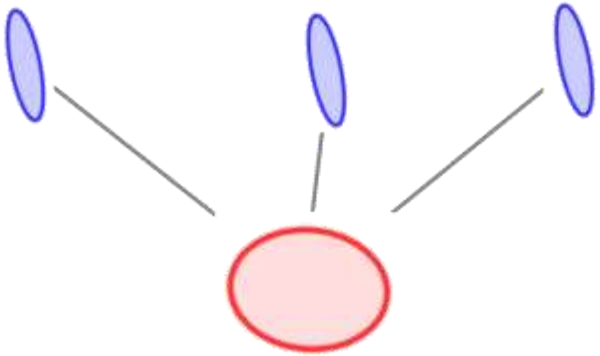
Penrose (*1931)

https://www.youtube.com/watch?v=JiDWGbsVEno

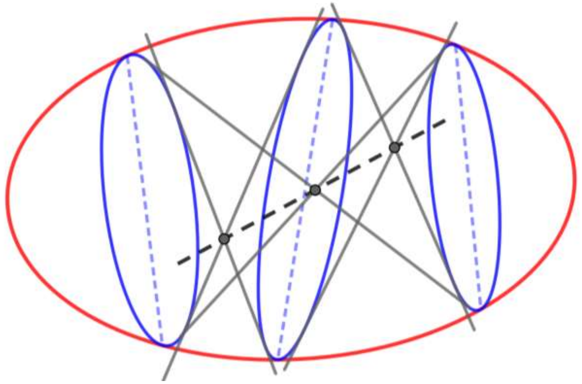
Penrose (1950)

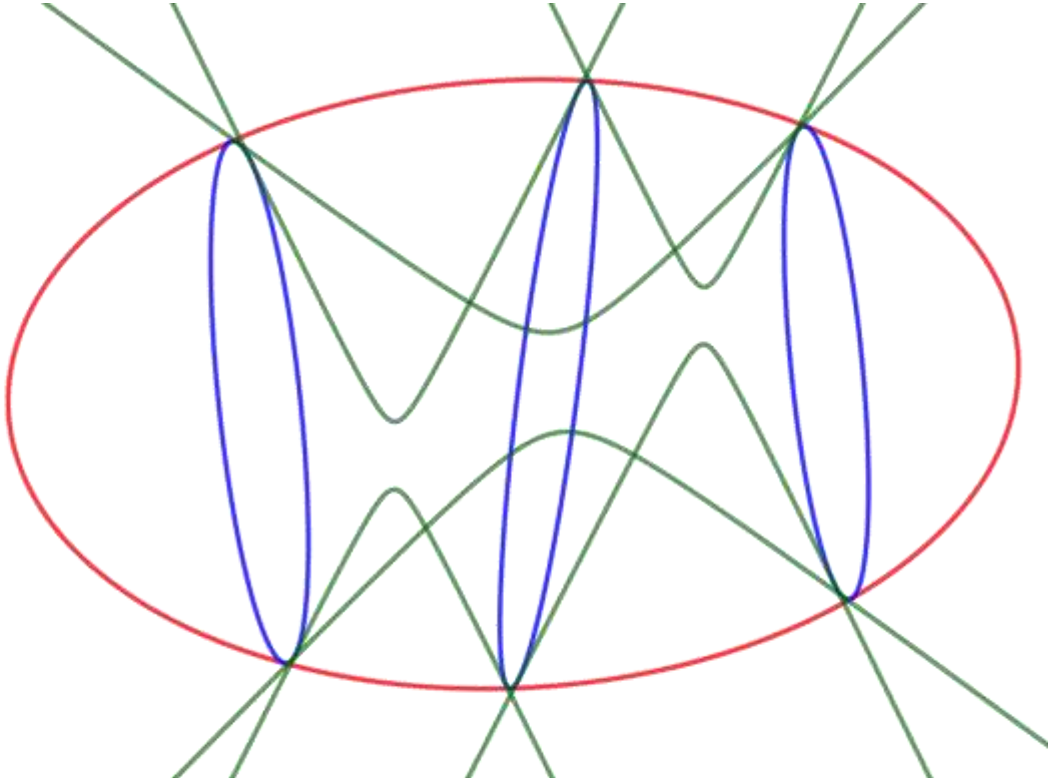


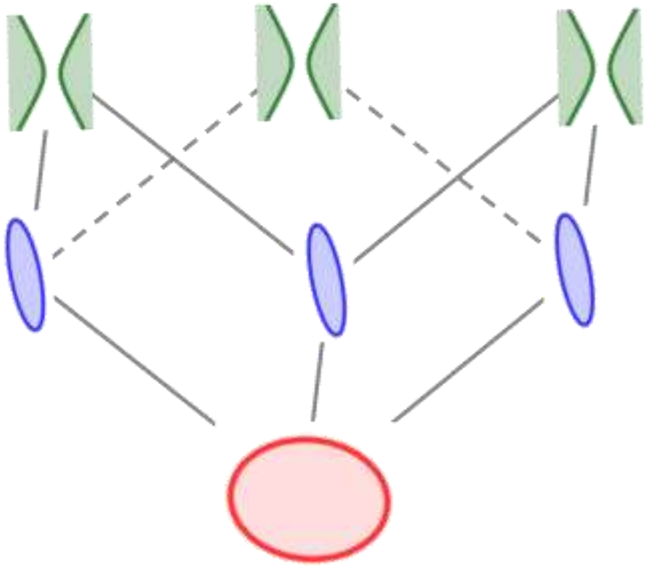




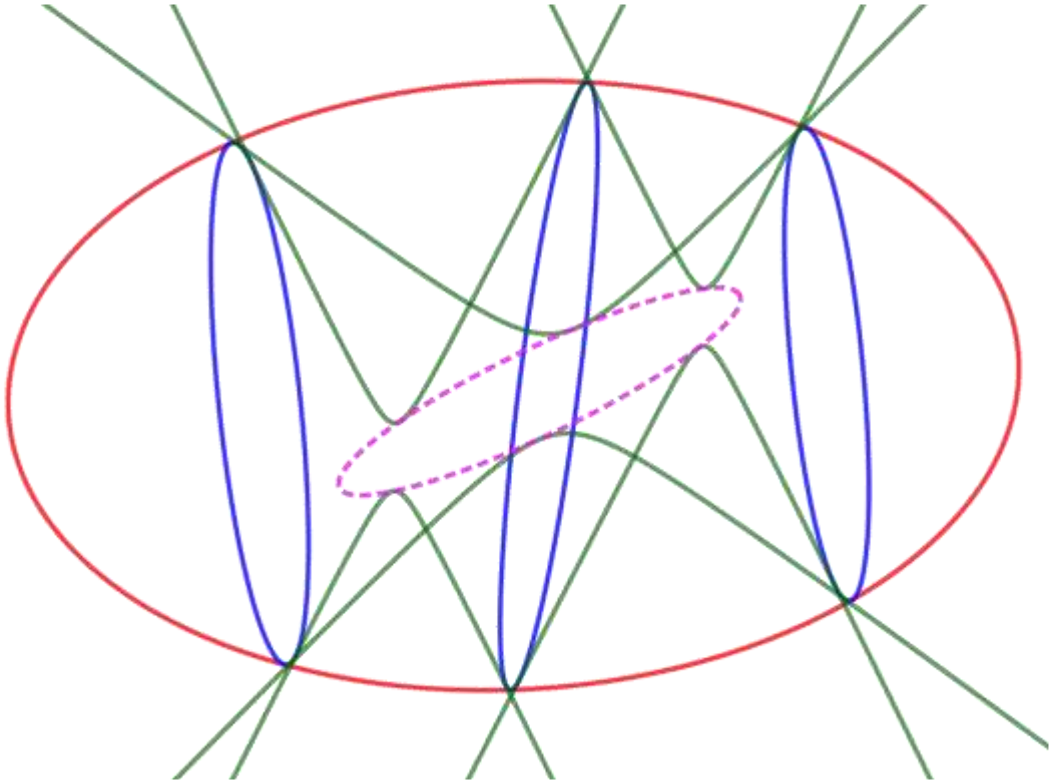
Penrose (1950)

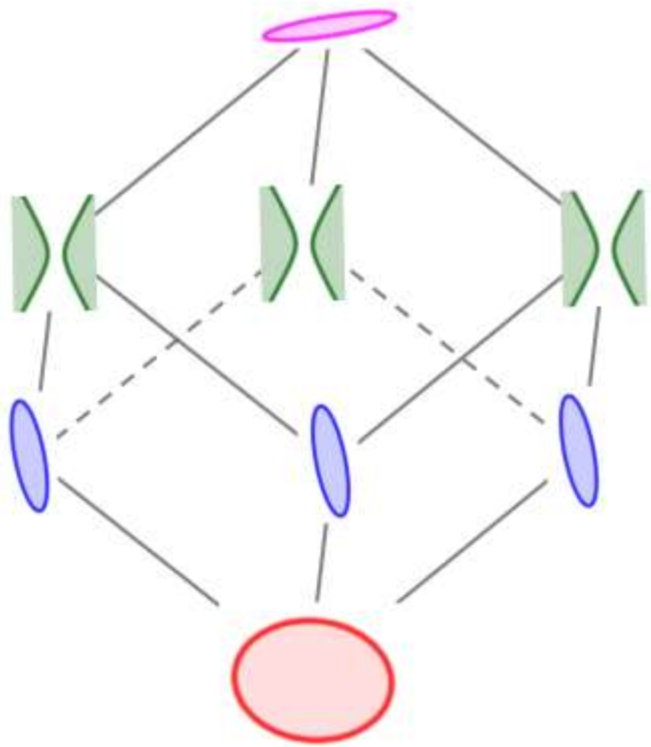






Penrose (1950)





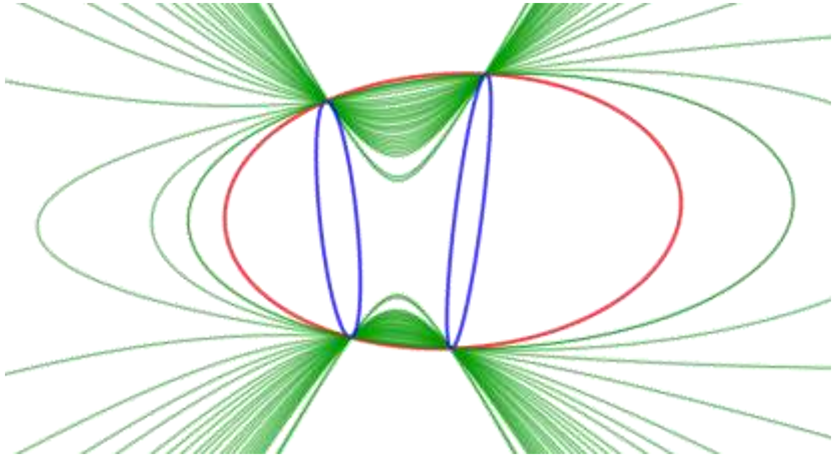
Penrose: Es seien 7 reguläre Kegelschnitte entsprechend den Ecken eines Würfels gegeben, so dass Kegelschnitte benachbarter Ecken Doppelkontakt miteinander haben (*).

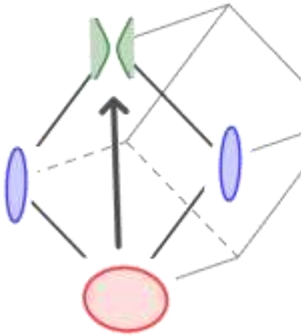
Dann gibt es einen 8. Kegelschnitt ebenfalls mit dieser Eigenschaft.

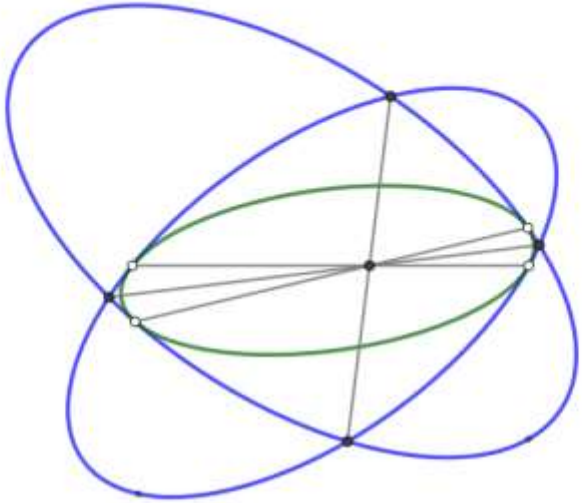
Penrose (1950)

Die Bedingung (*):

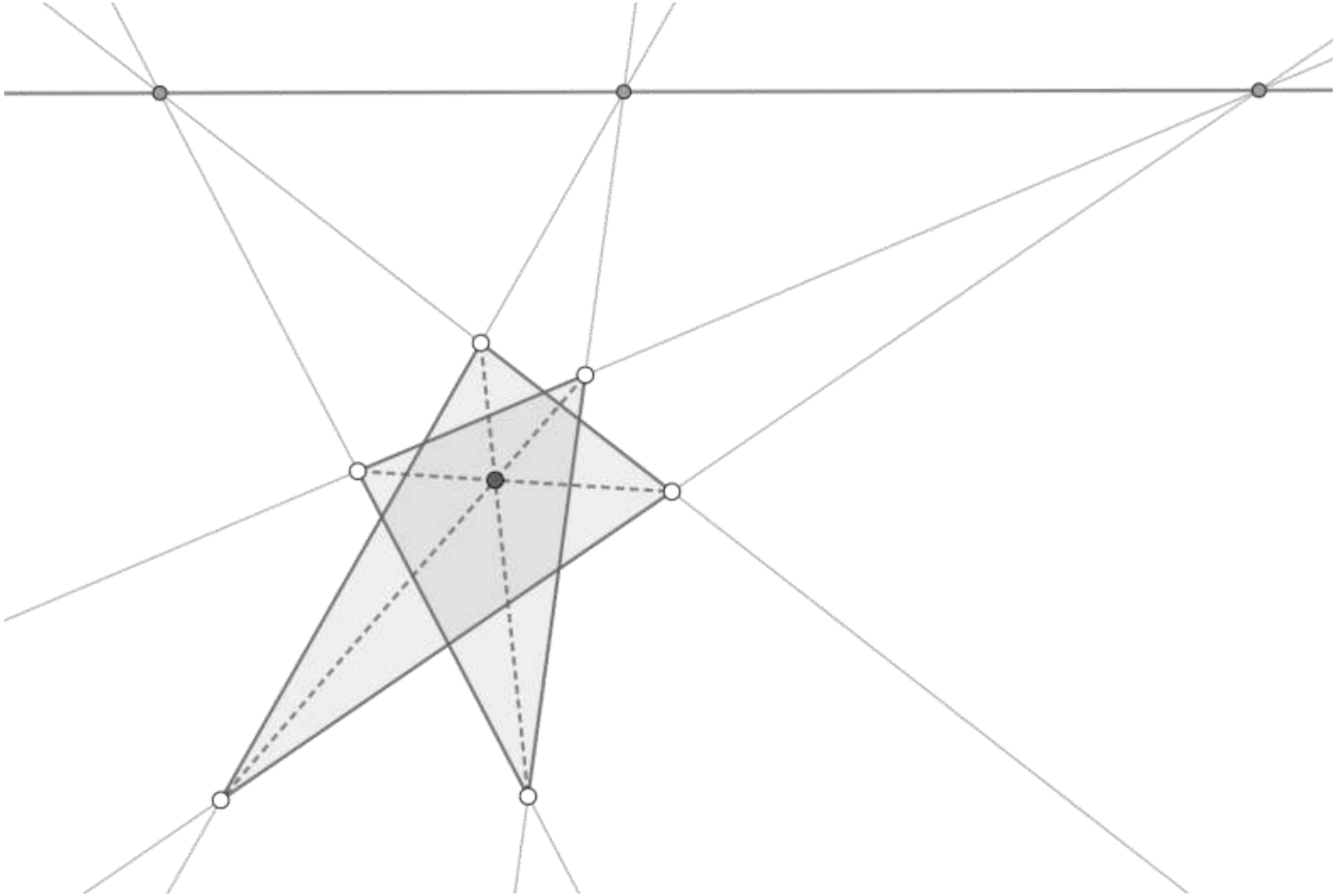
Gegenüberliegende Kegelschnitte in einer Würfelfläche
müssen sich kontinuierlich ineinander verwandeln lassen.



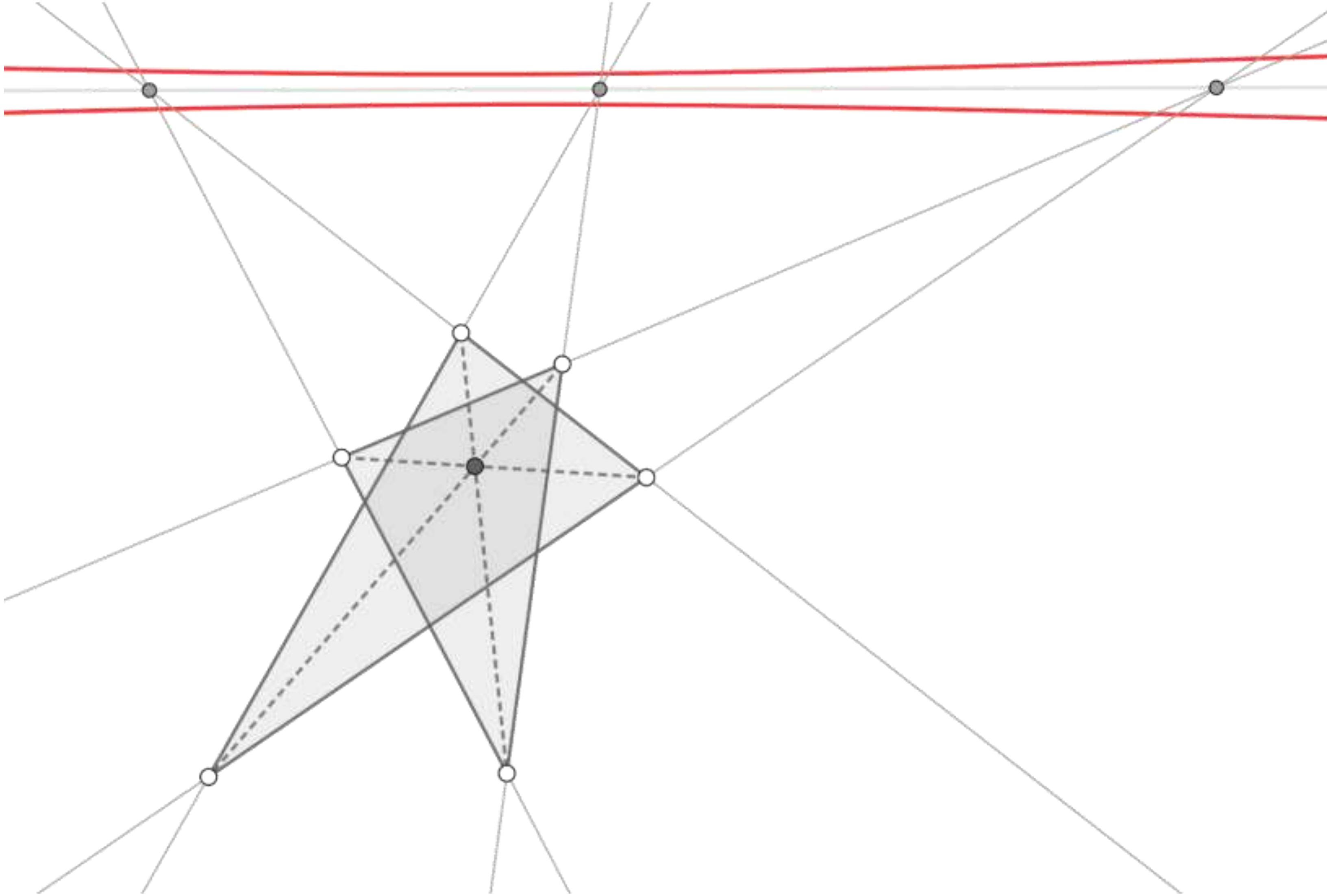


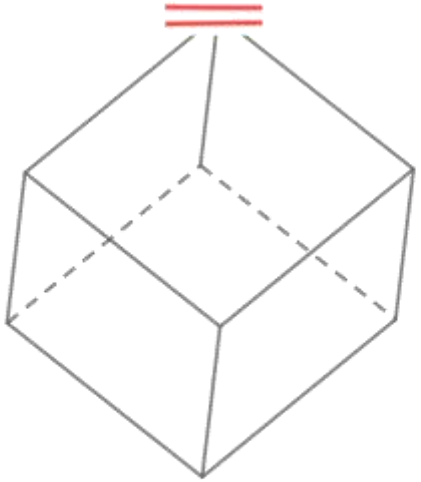


Desargues -> Penrose

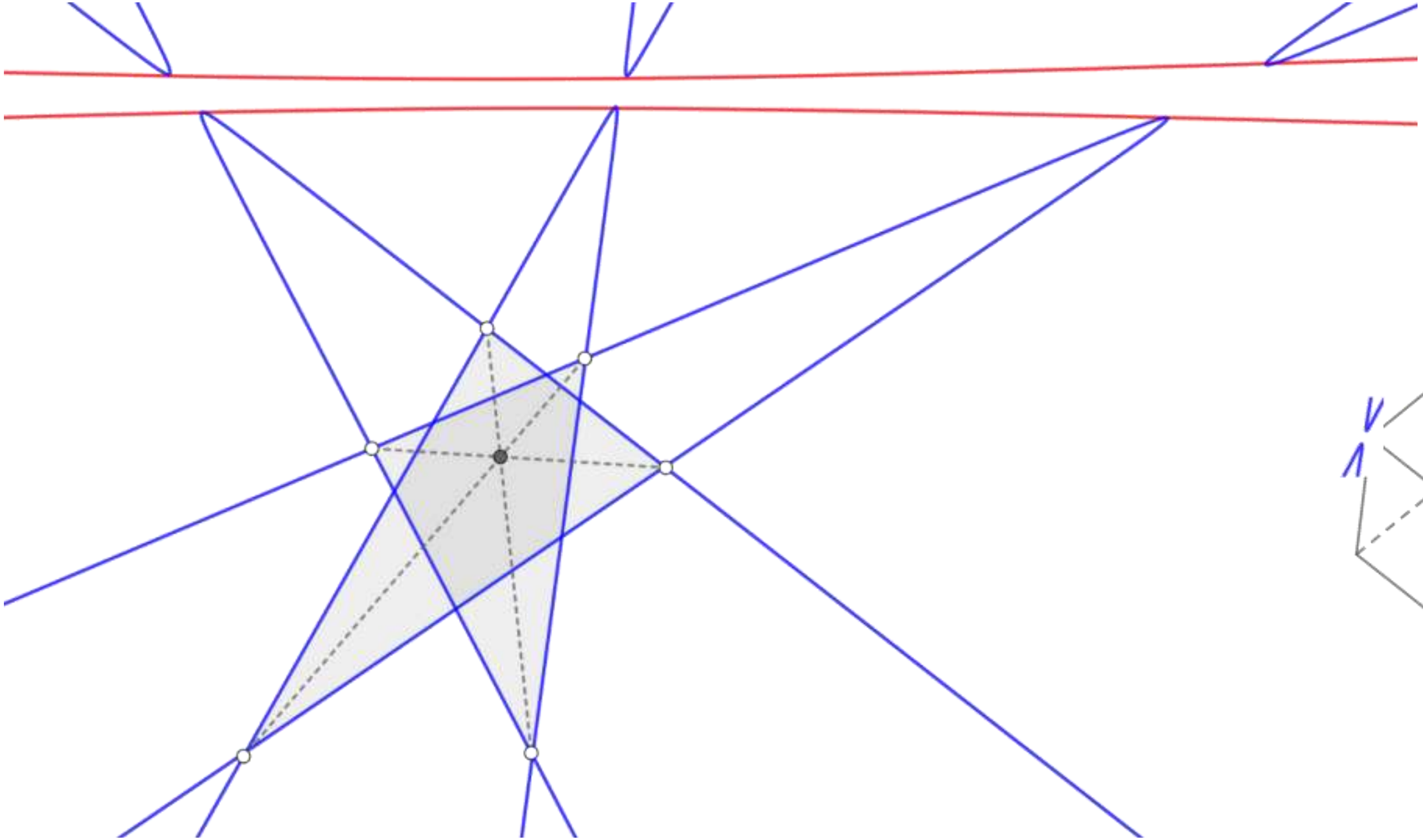


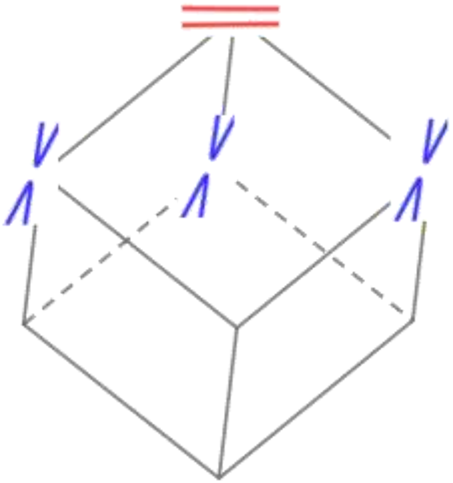
Desargues -> Penrose



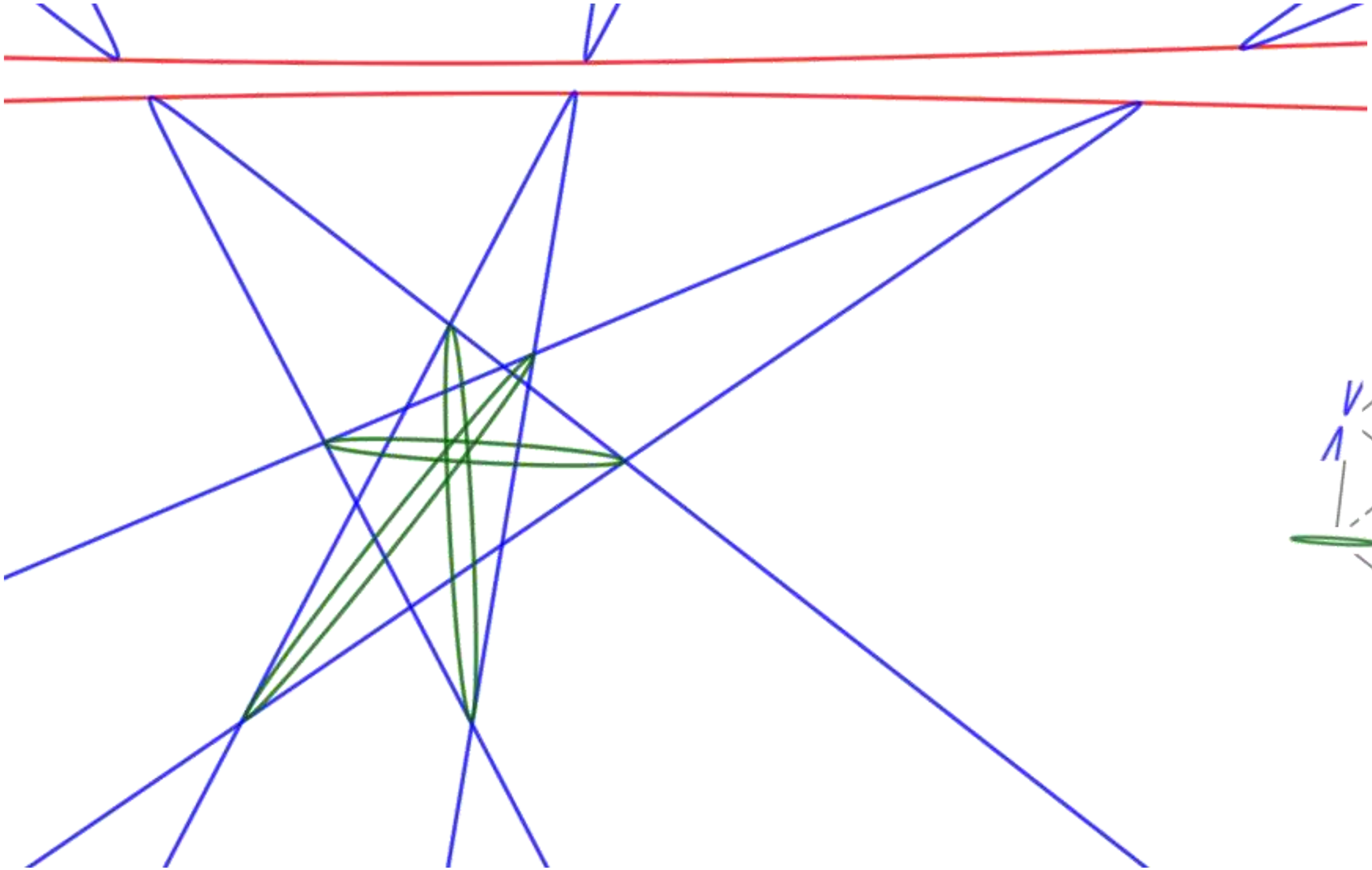


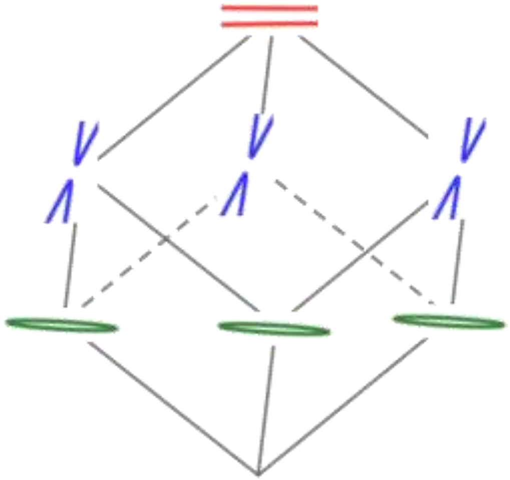
Desargues -> Penrose



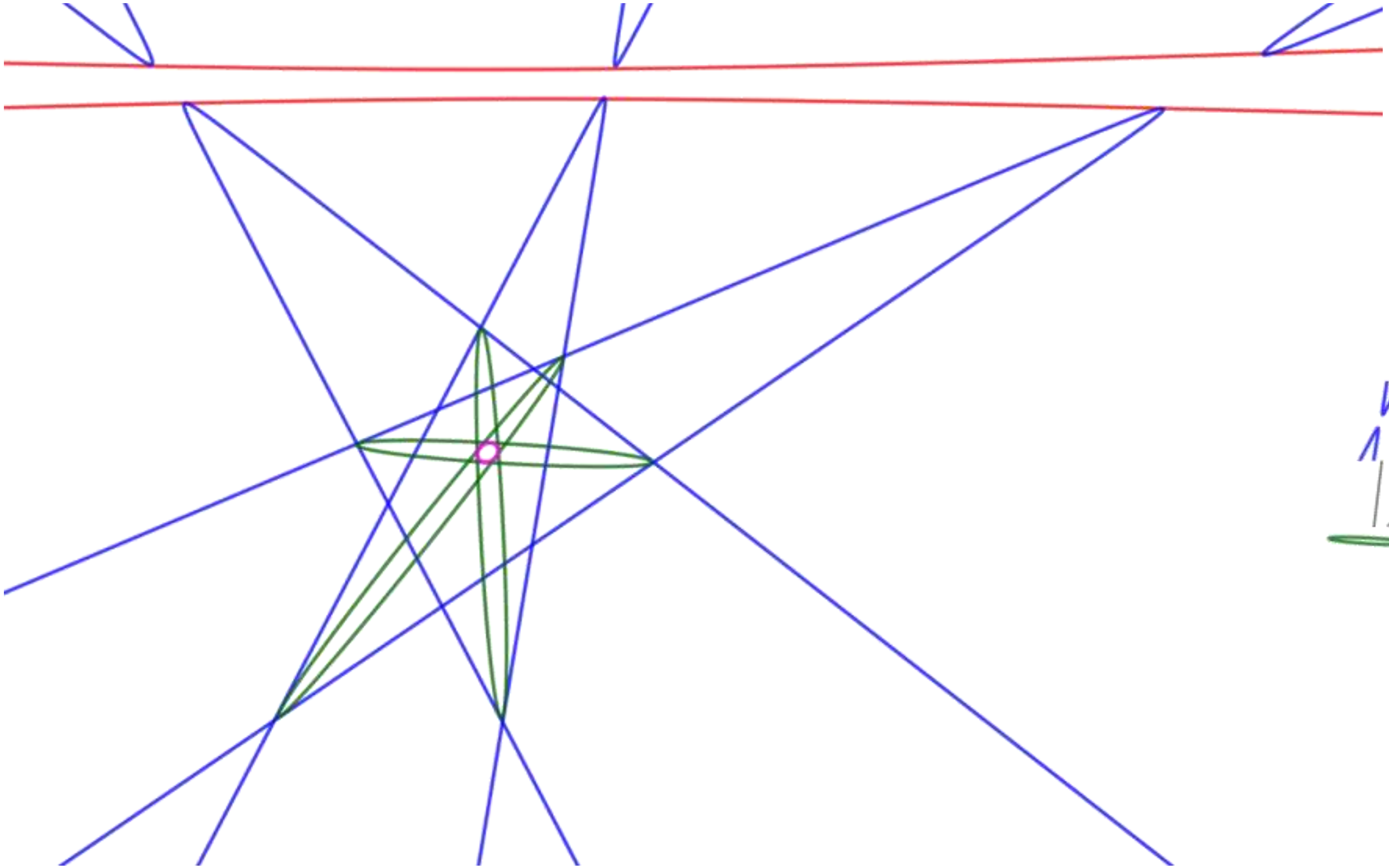


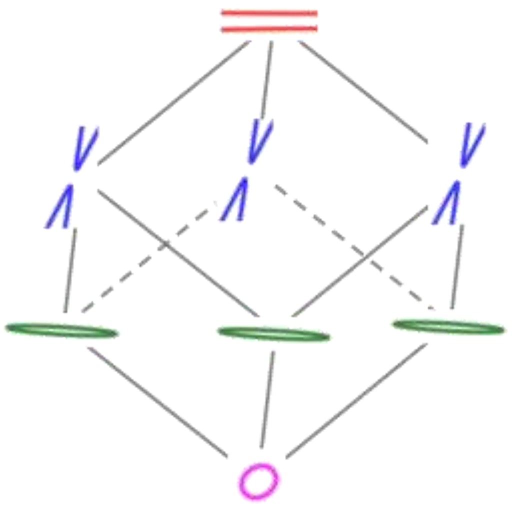
Desargues -> Penrose



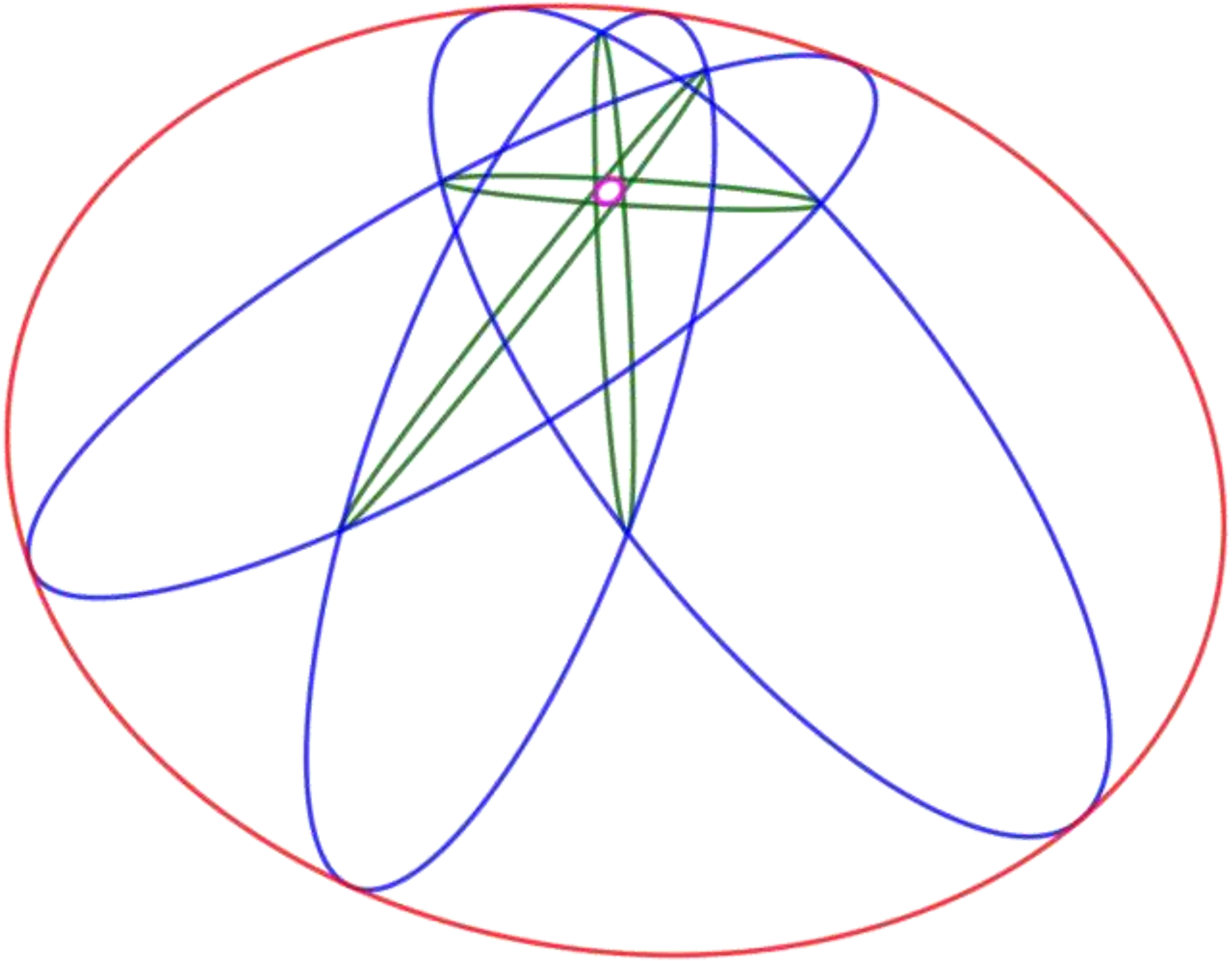


Desargues -> Penrose

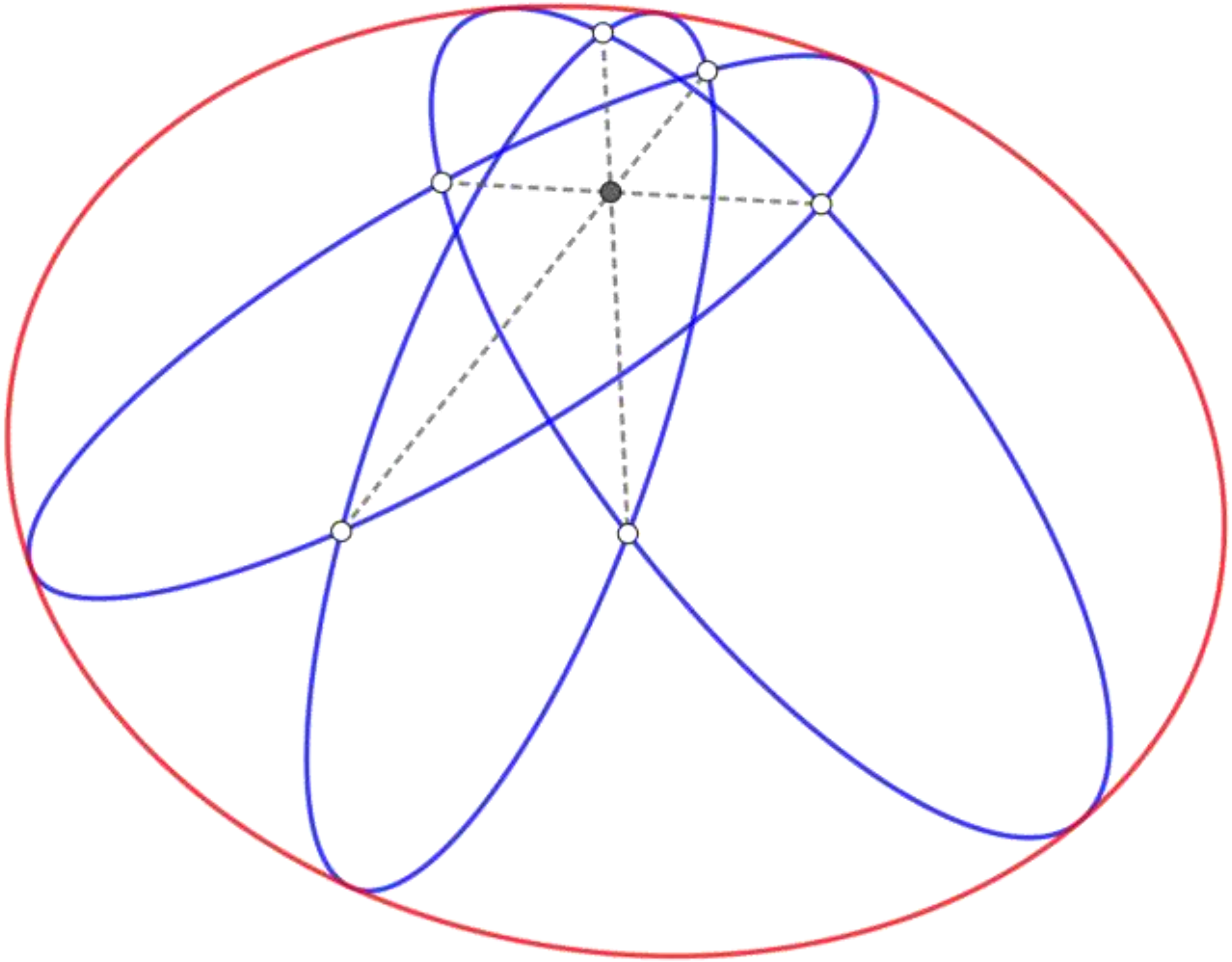




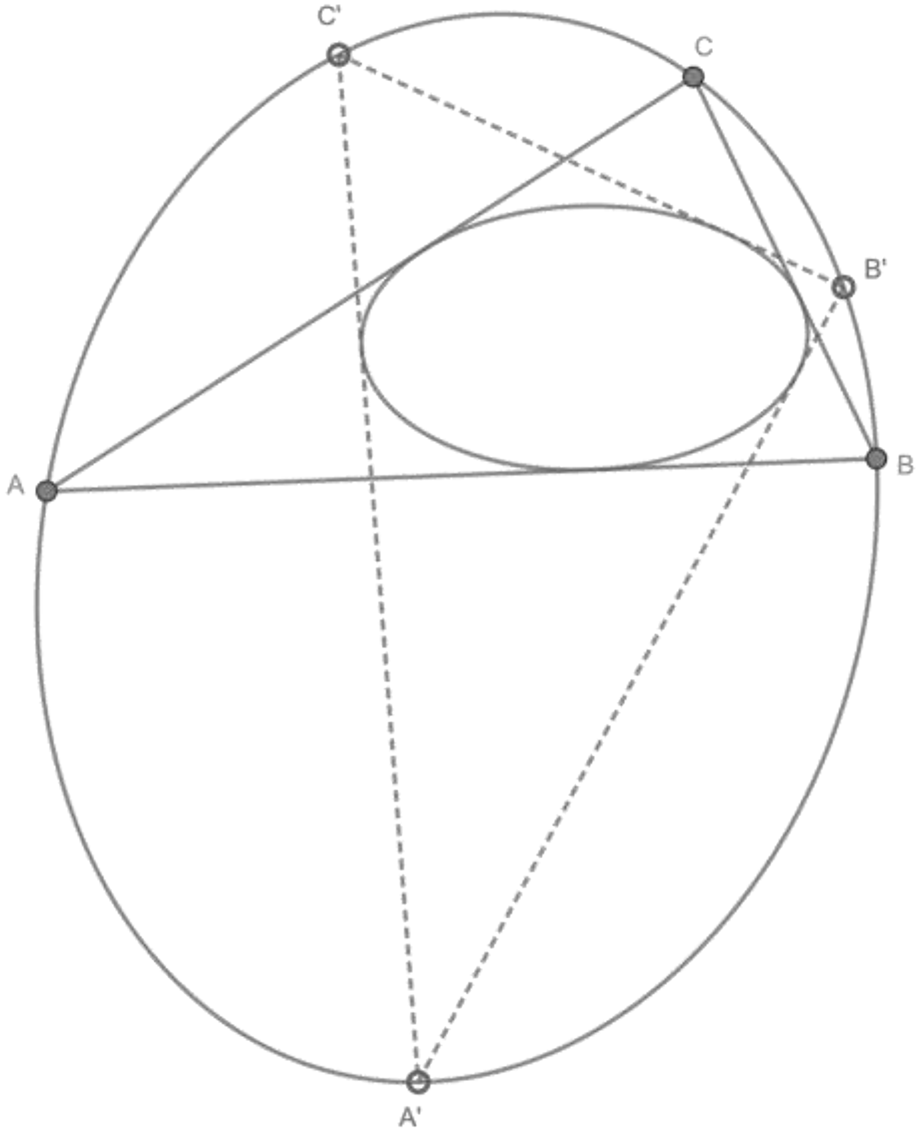
Desargues -> Penrose



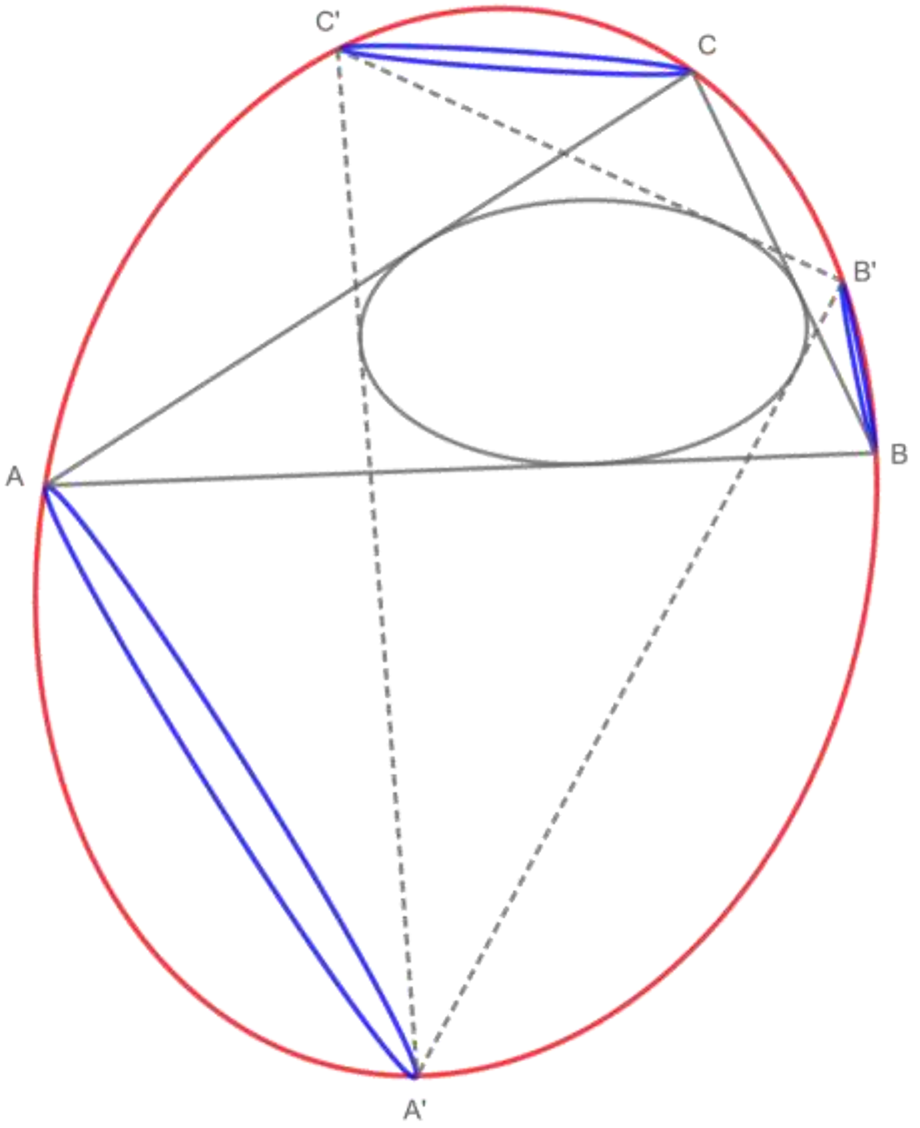
Desargues -> Penrose -> Salmon

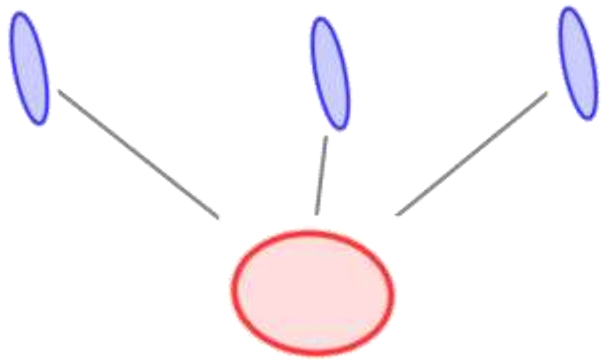


Poncelet -> Penrose

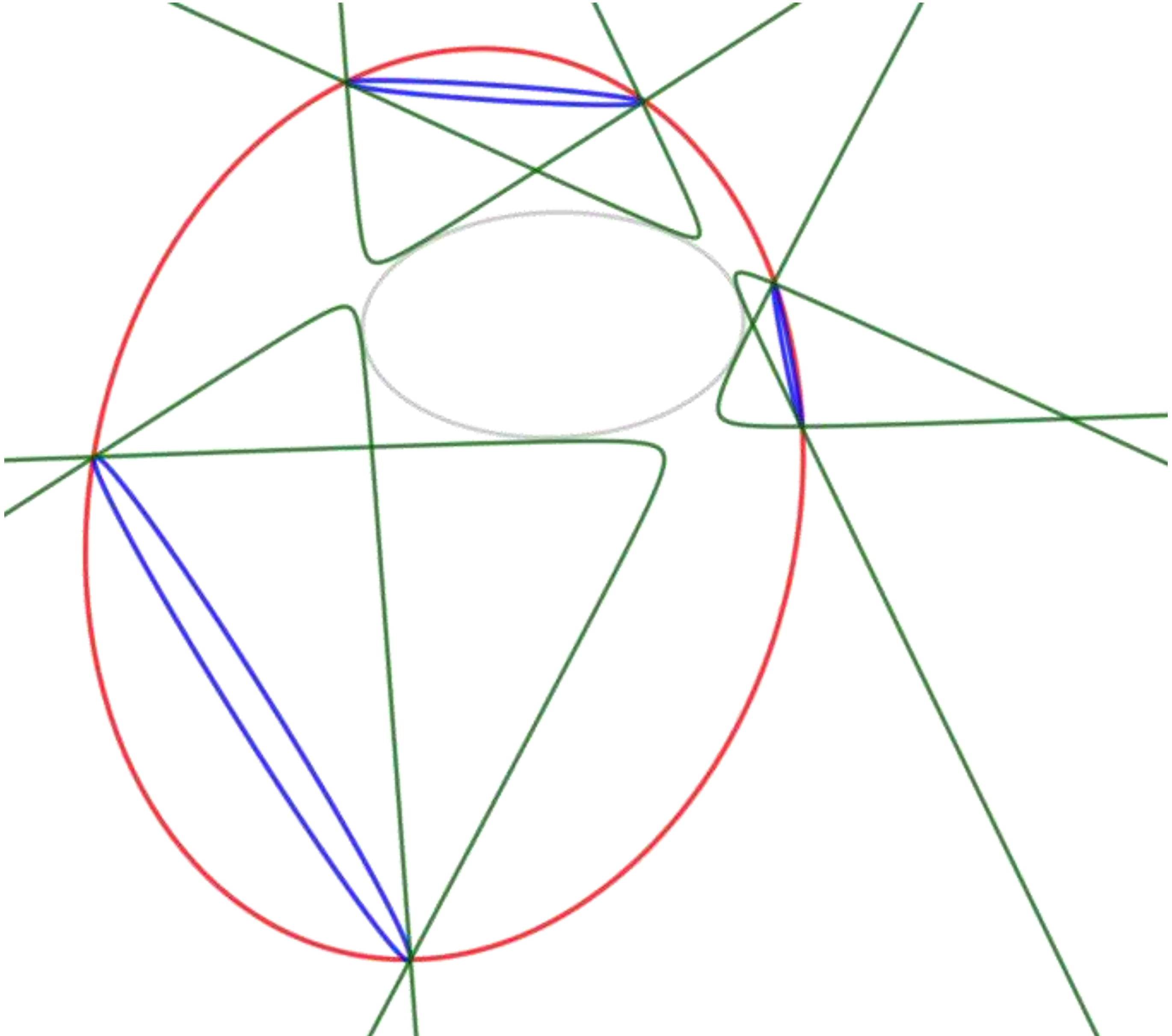


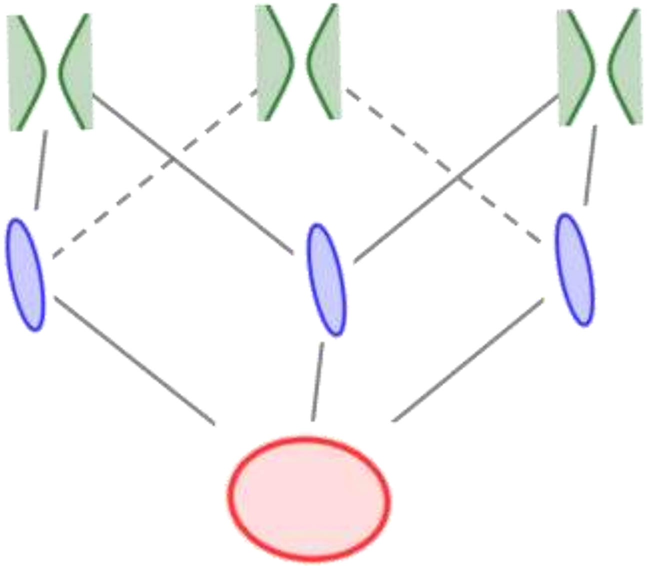
Poncelet -> Penrose



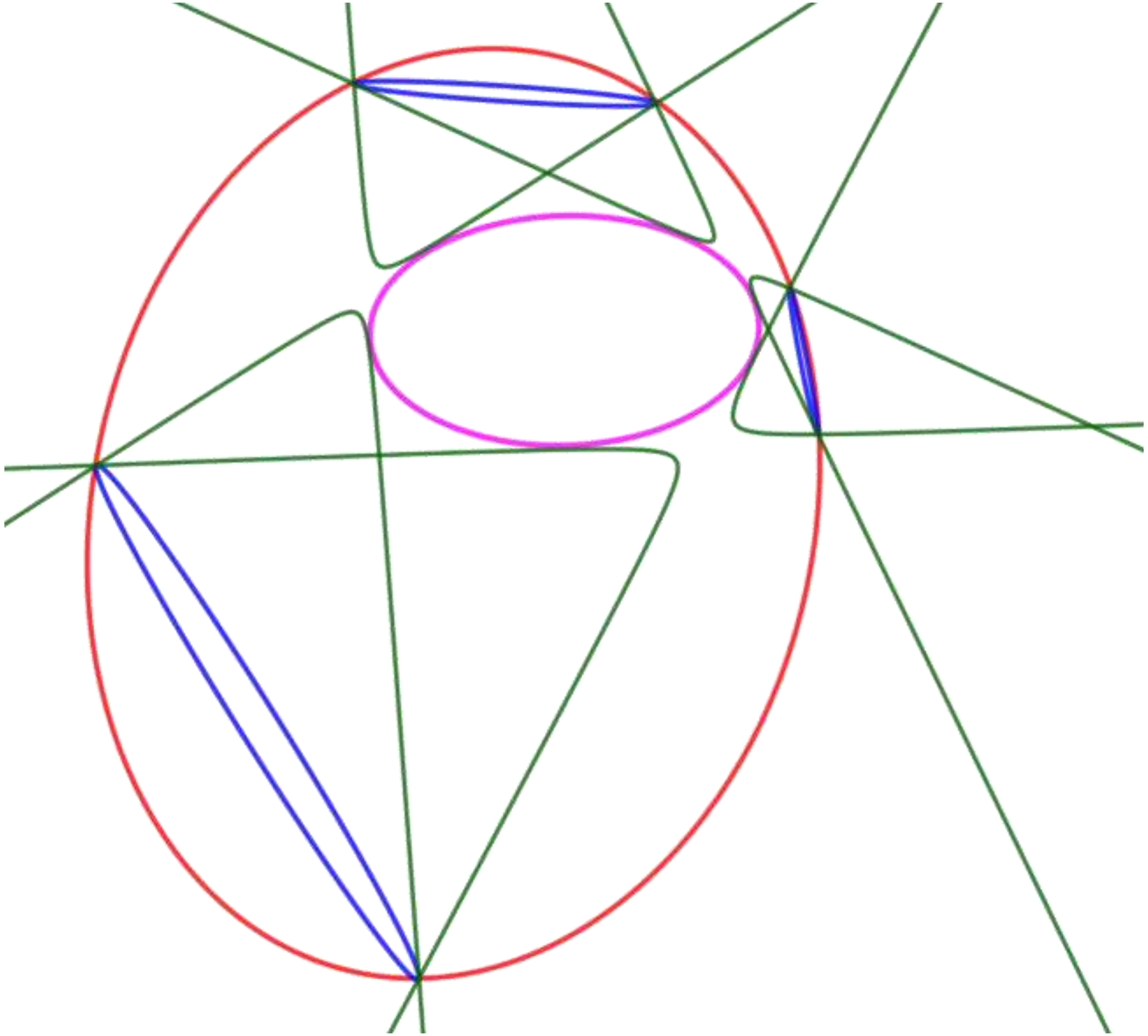


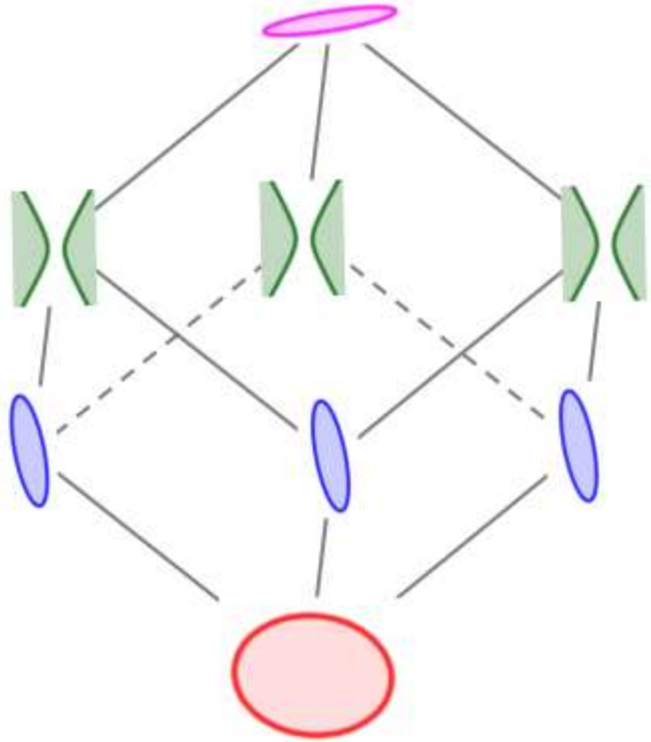
Poncelet -> Penrose



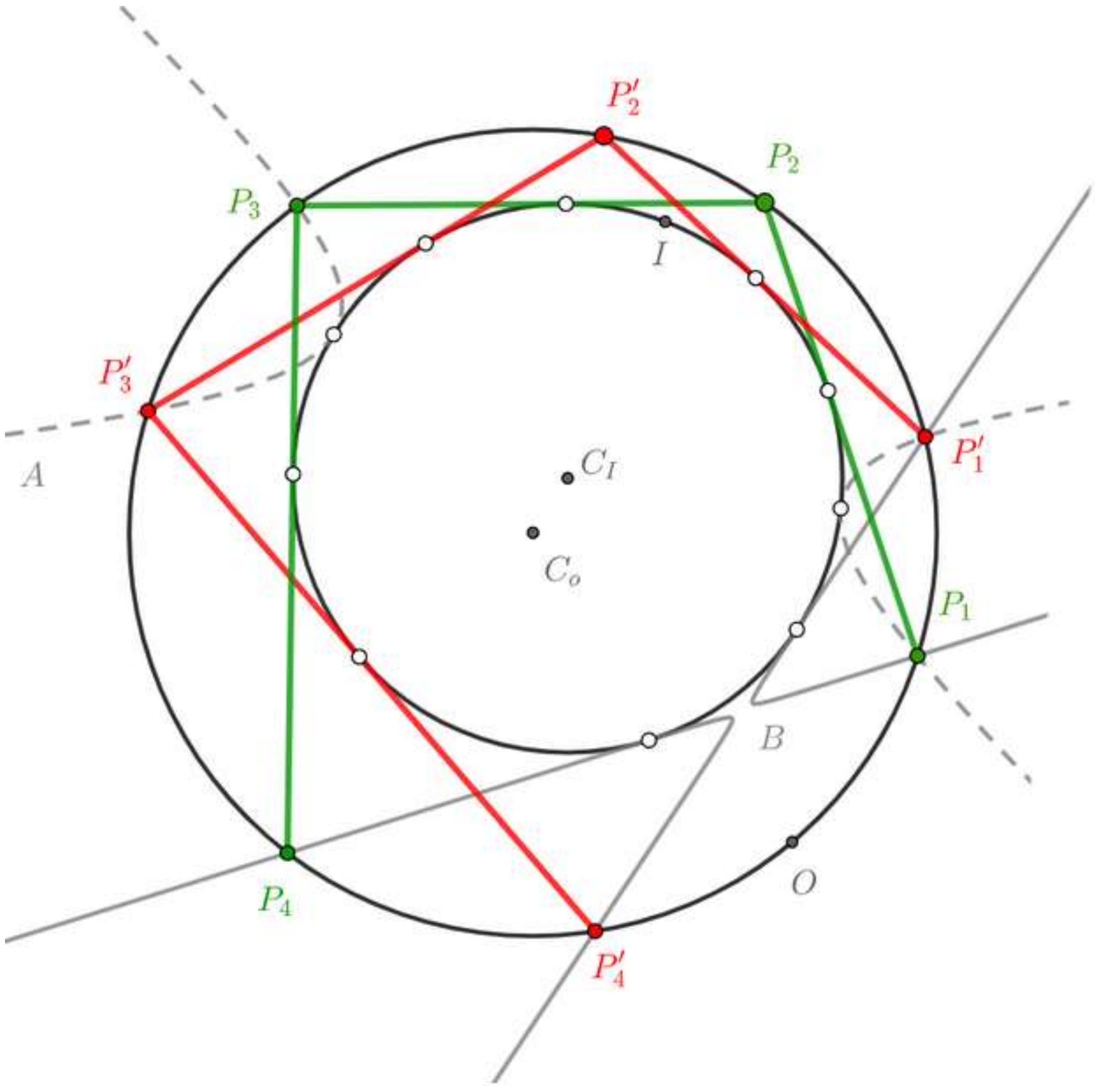


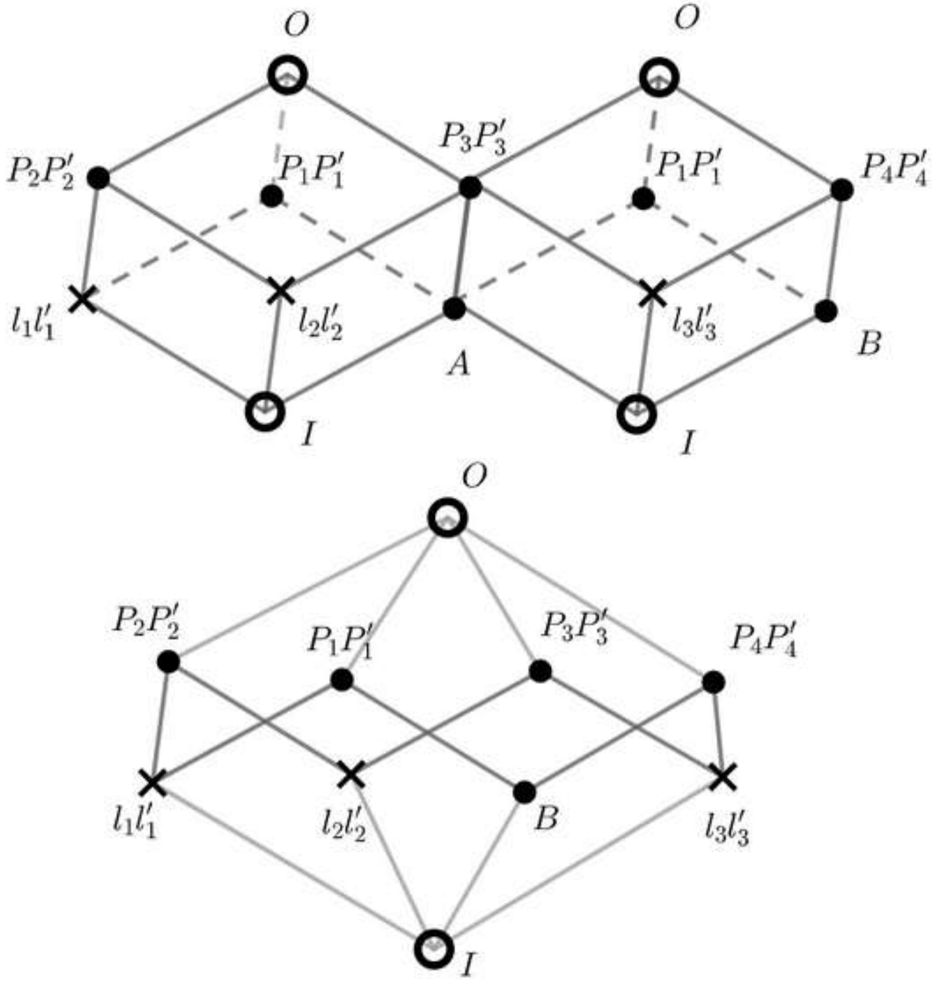
Poncelet -> Penrose



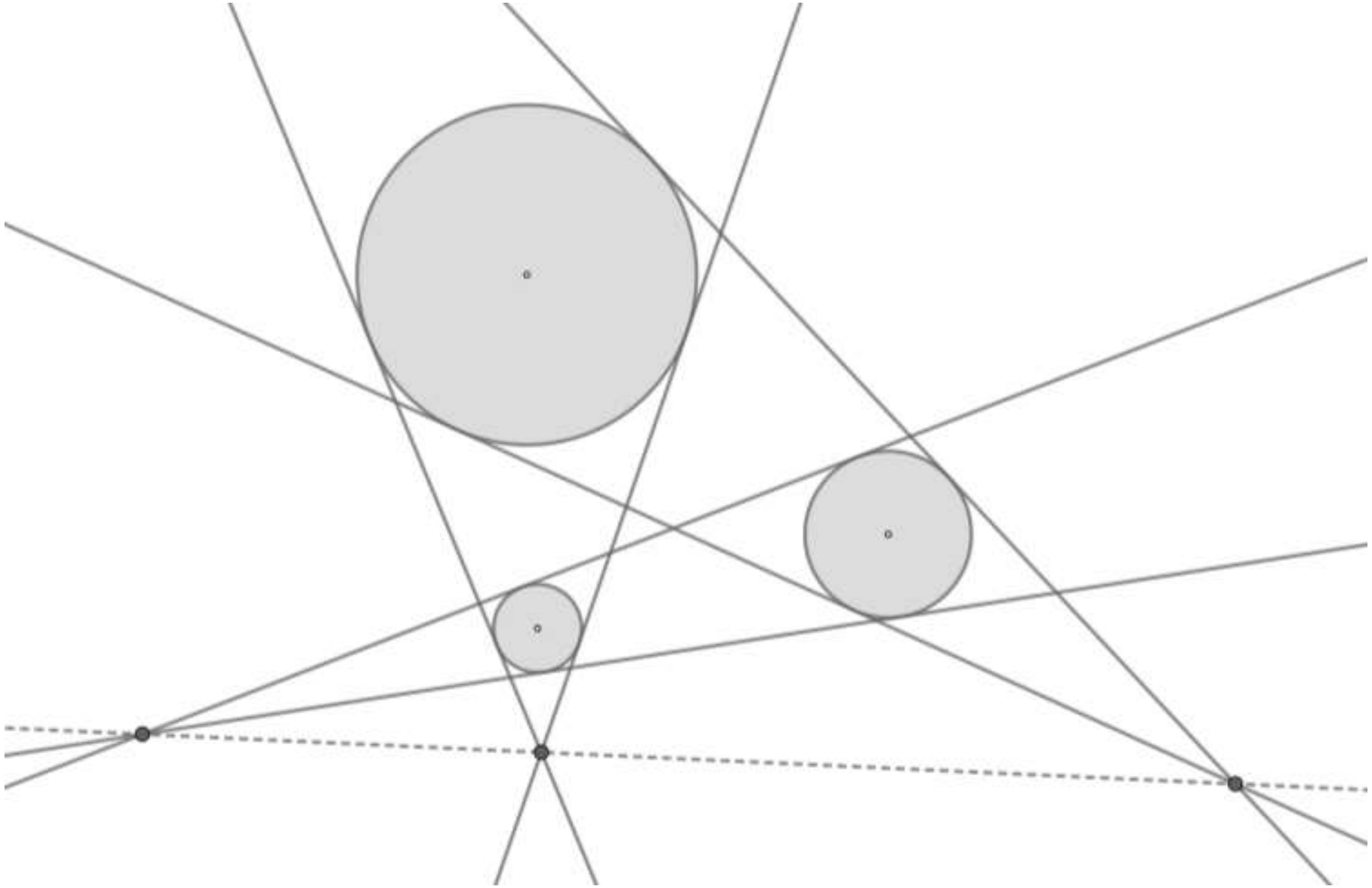


Poncelet für 4-Ecke

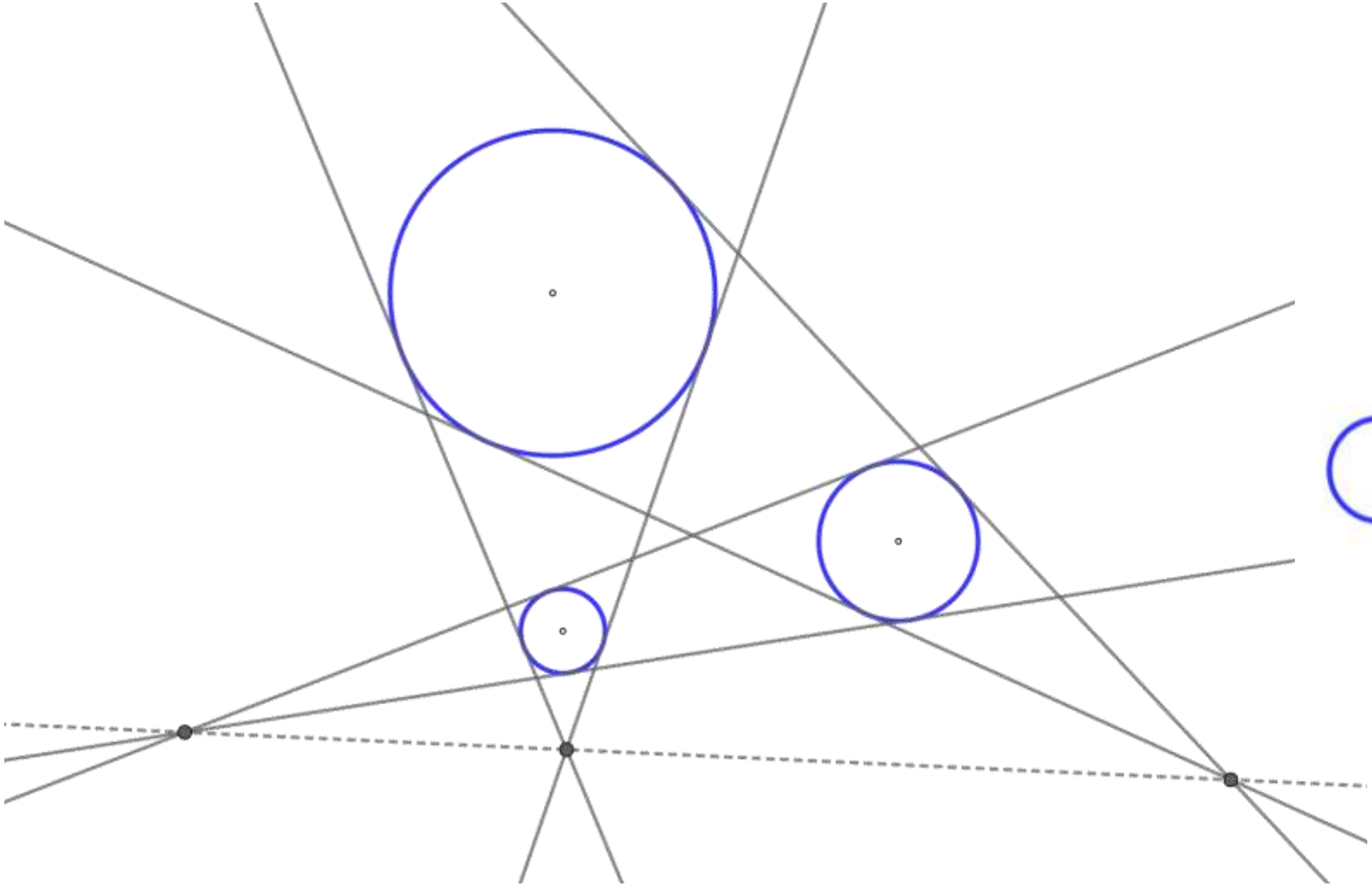


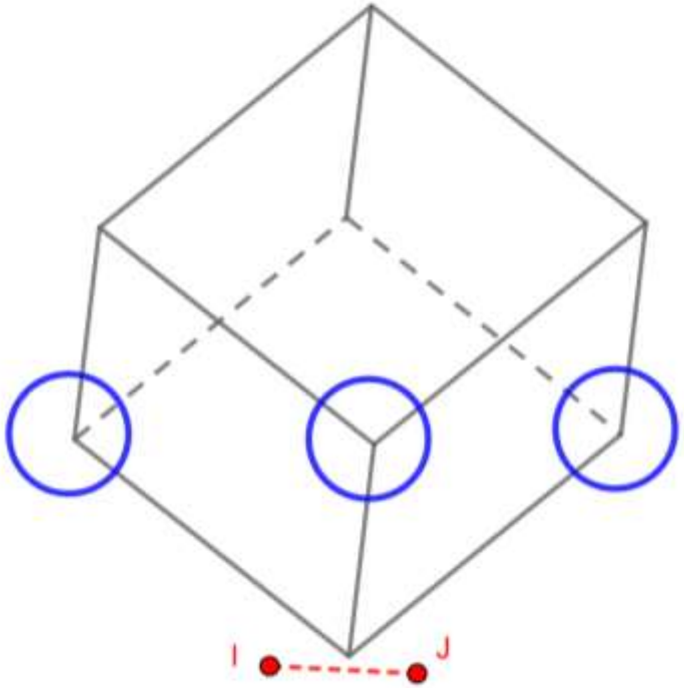


Monge -> Penrose

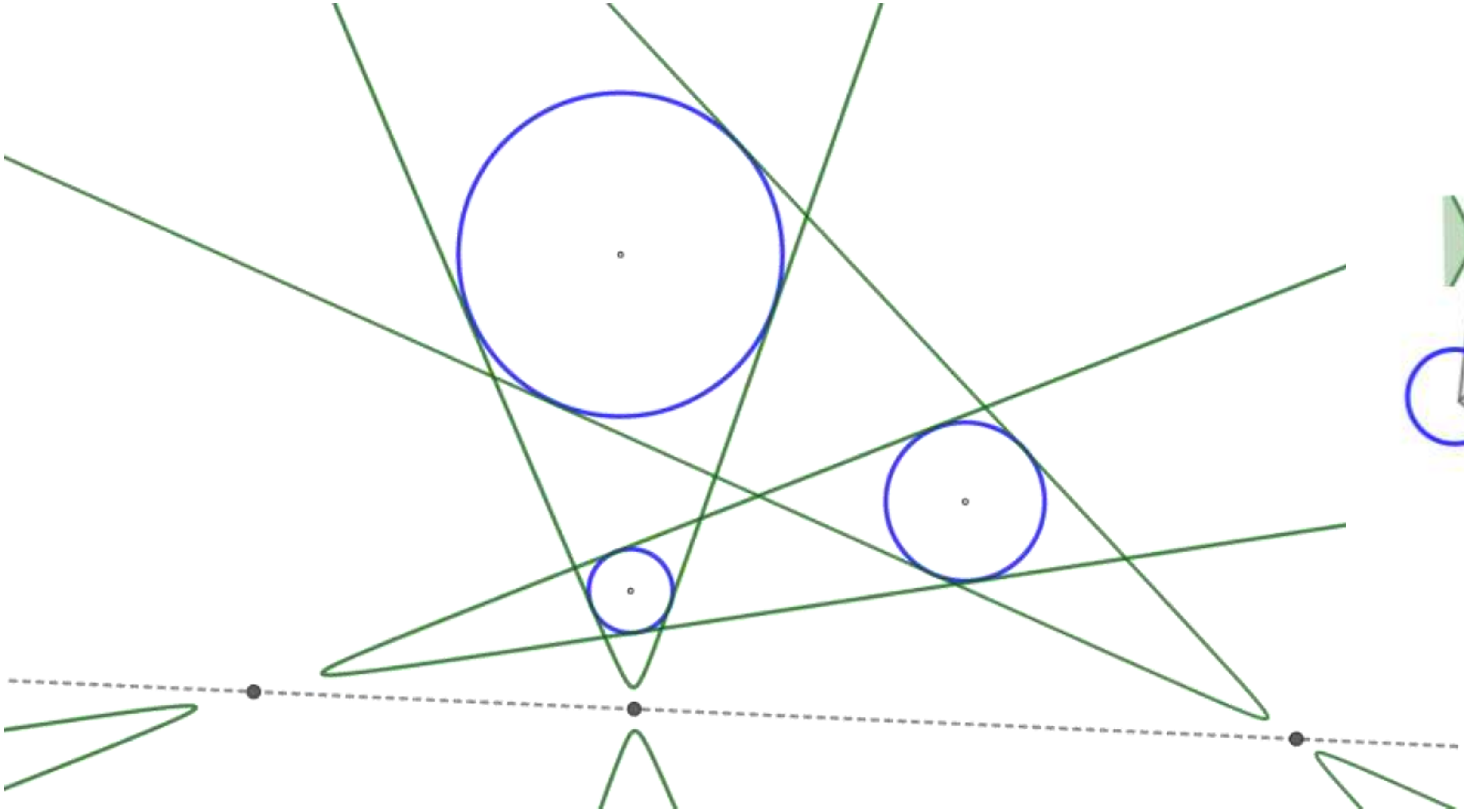


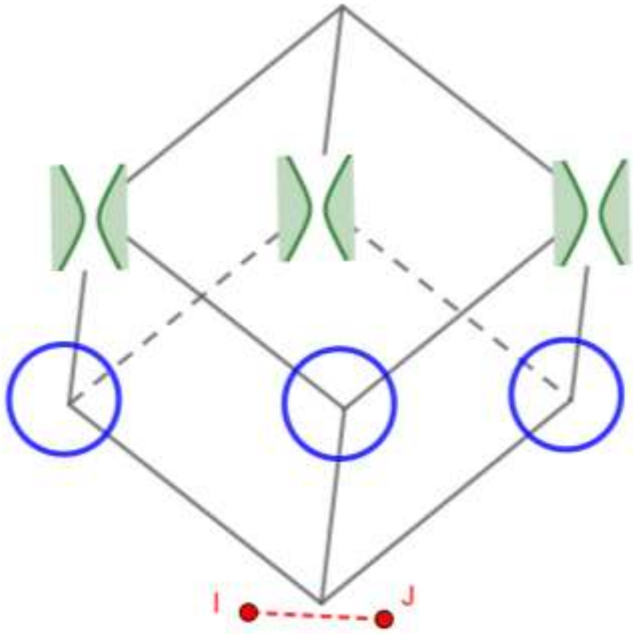
Monge -> Penrose



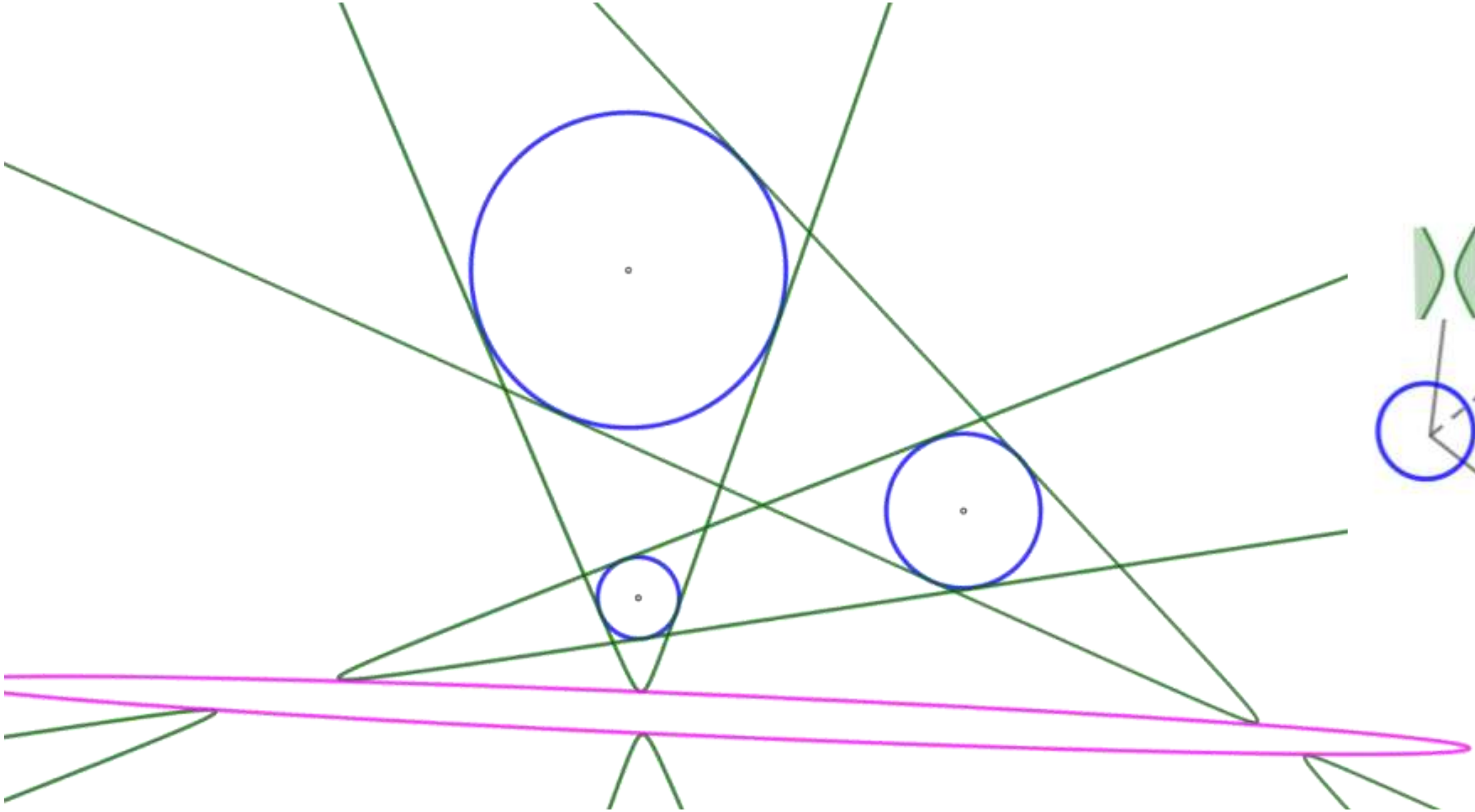


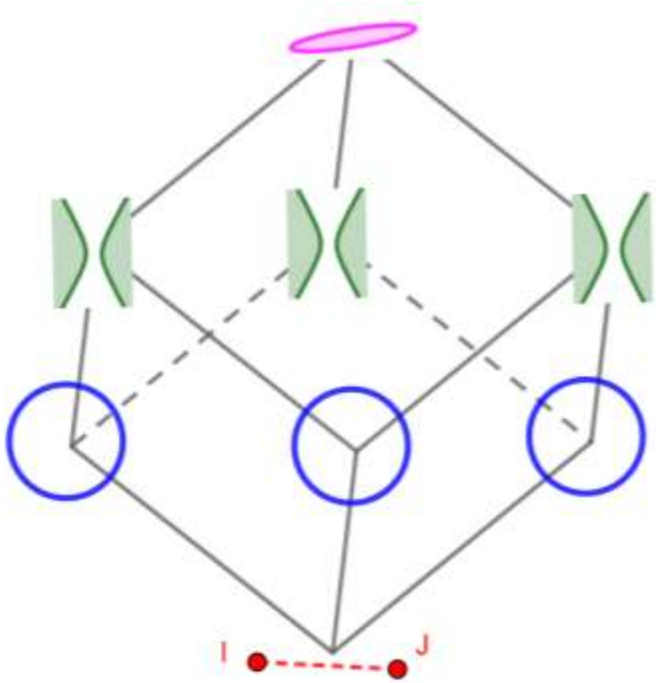
Monge -> Penrose



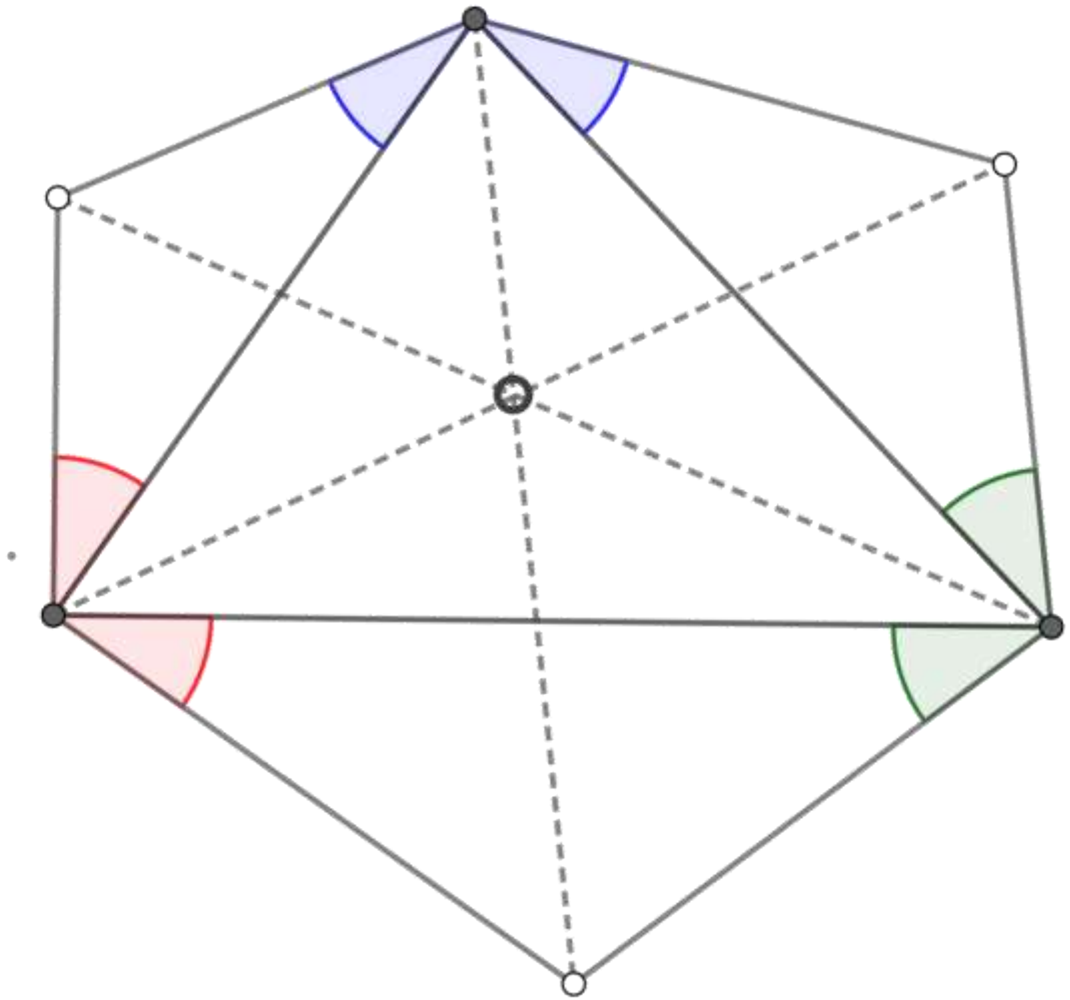


Monge -> Penrose

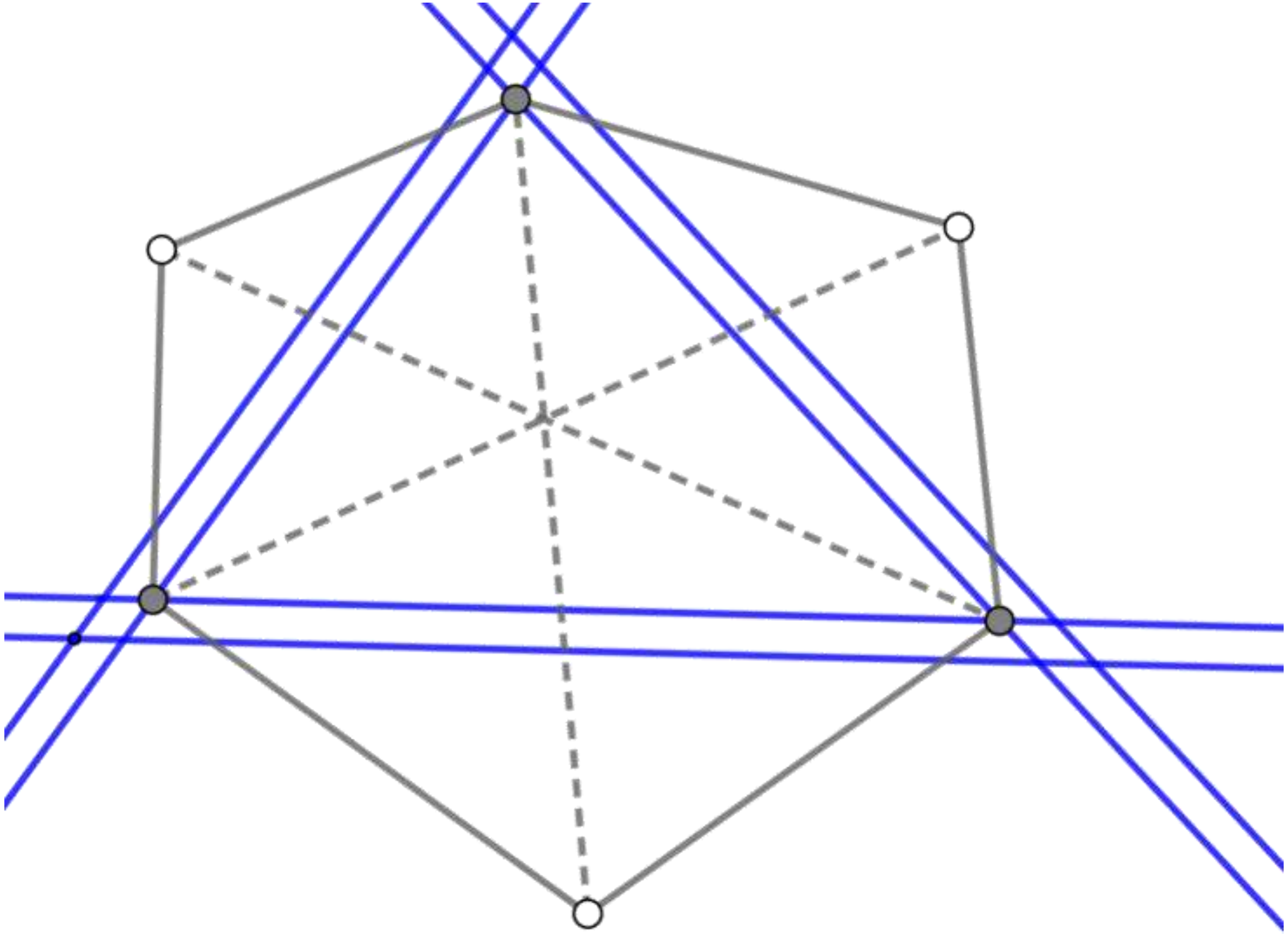


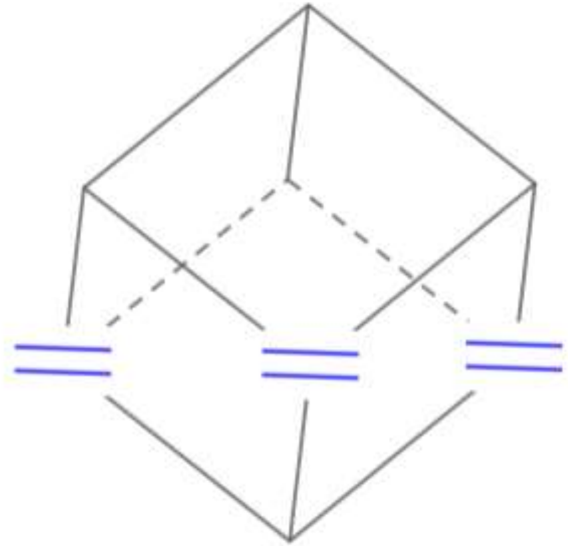


Jacobi (1825) / Escher (1923)-> Penrose

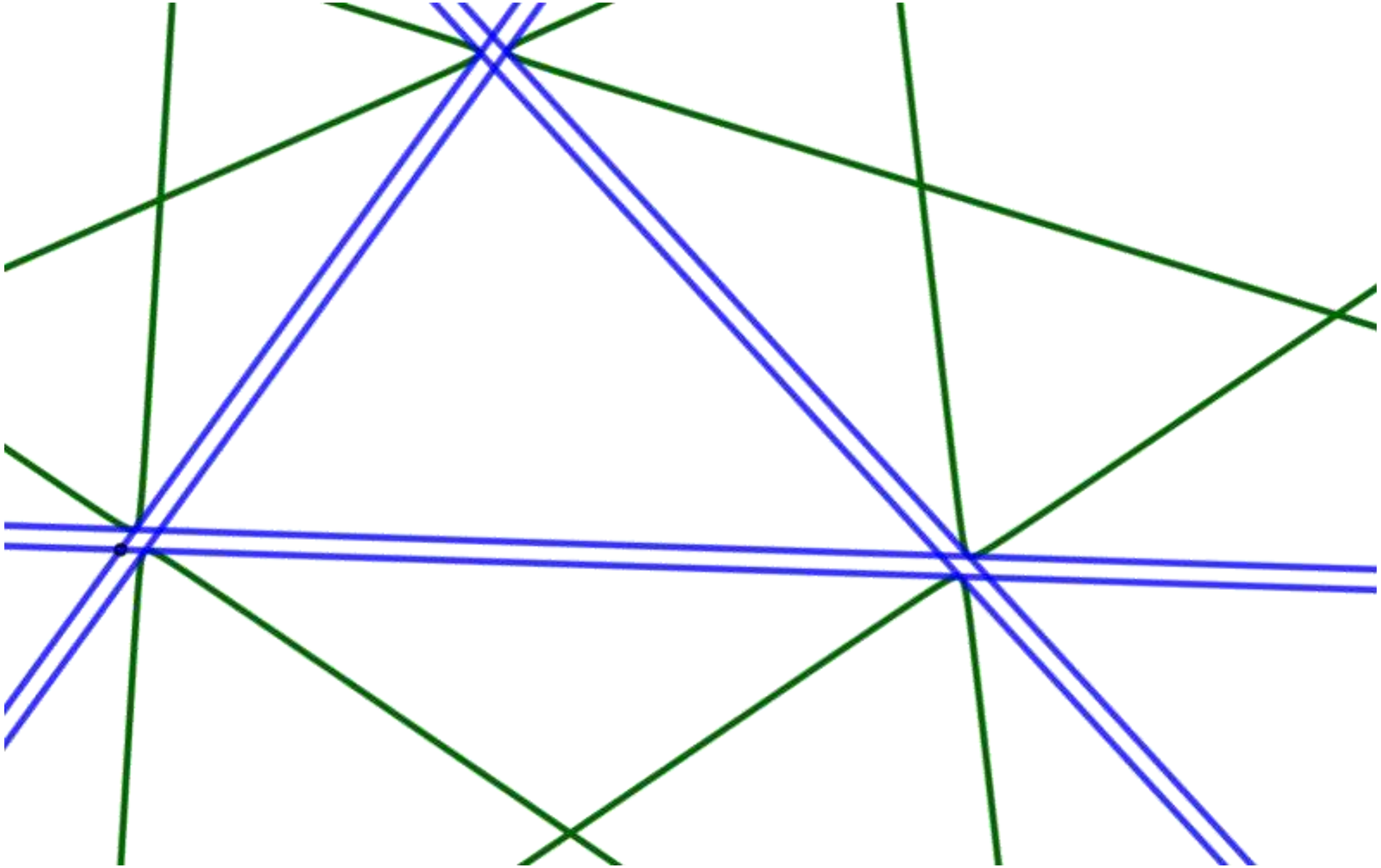


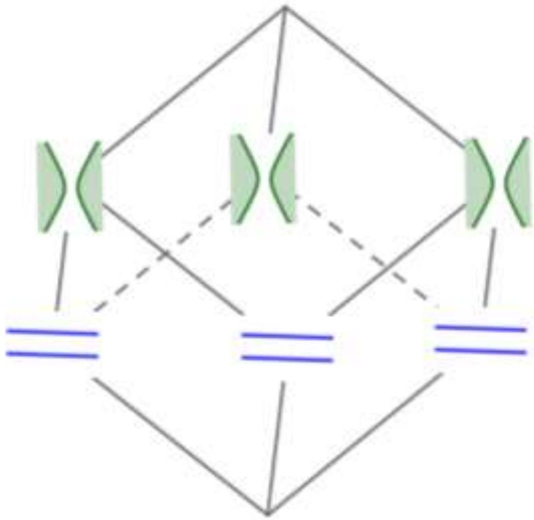
Jacobi (1825) / Escher (1923)-> Penrose



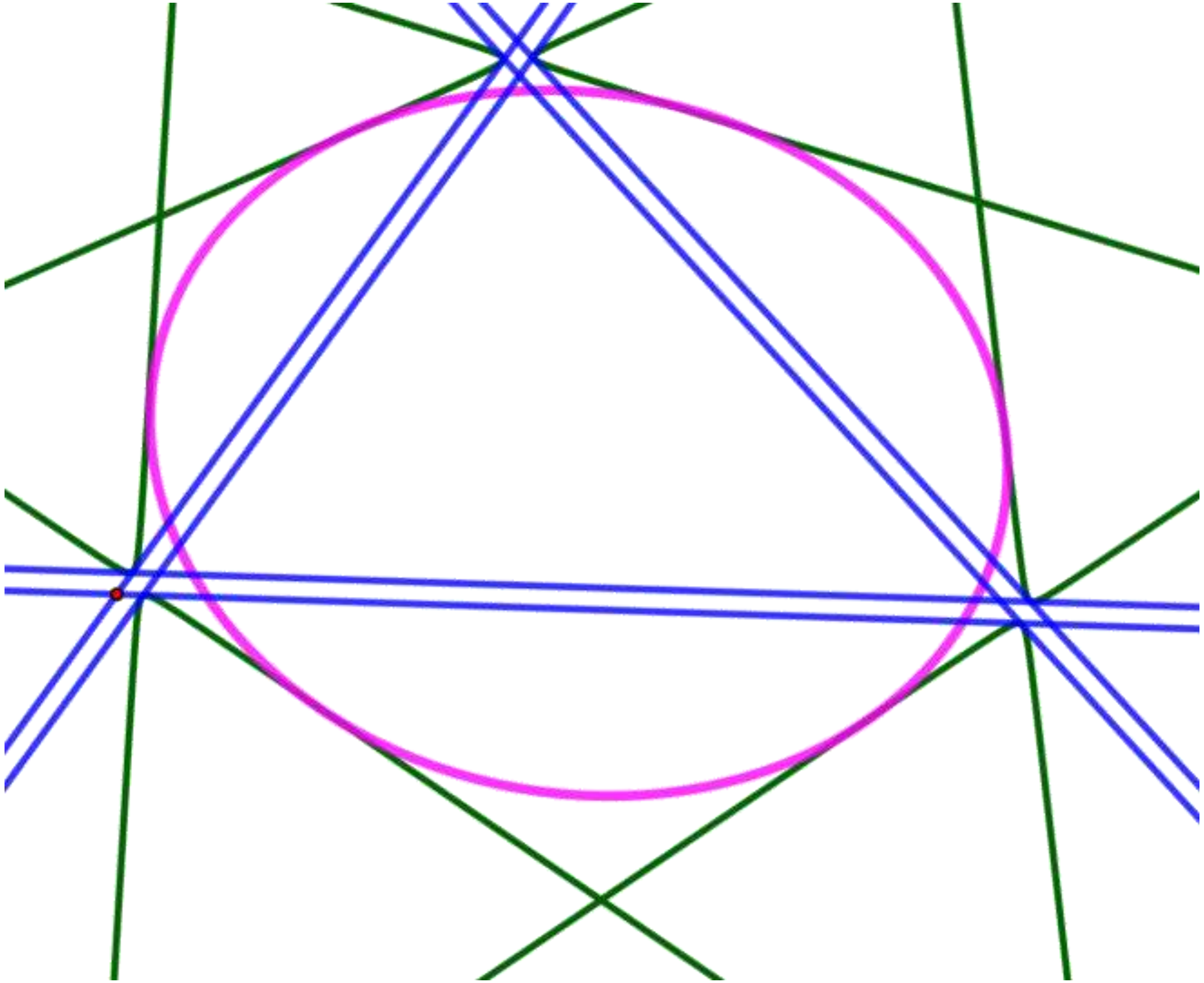


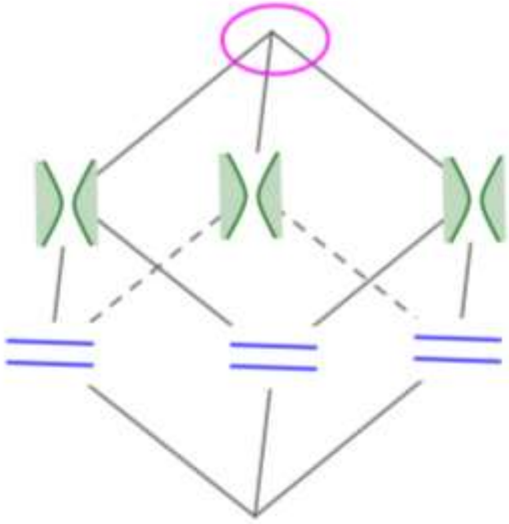
Jacobi (1825) / Escher ()-> Penrose



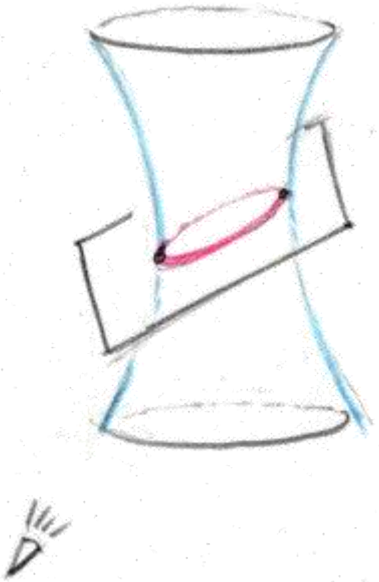


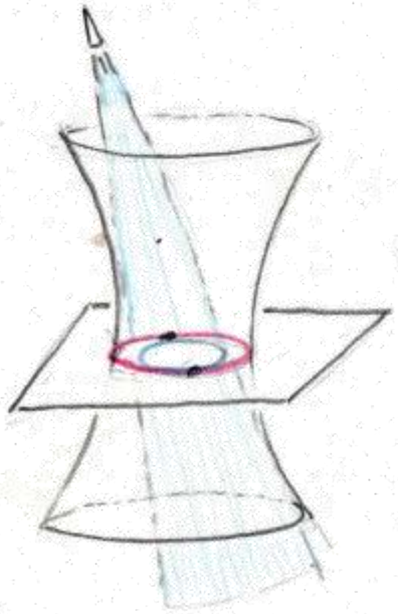
Jacobi (1825) / Escher (1923)-> Penrose



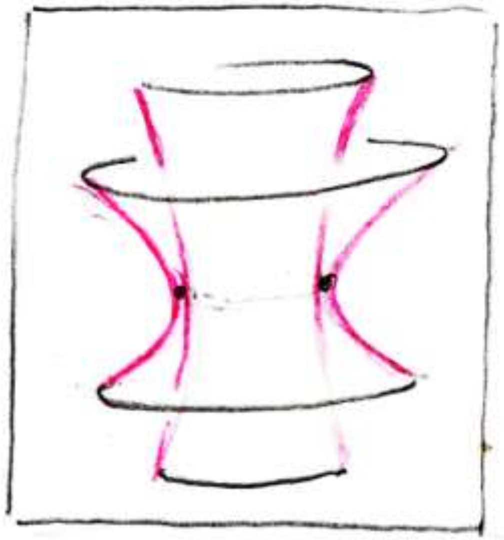


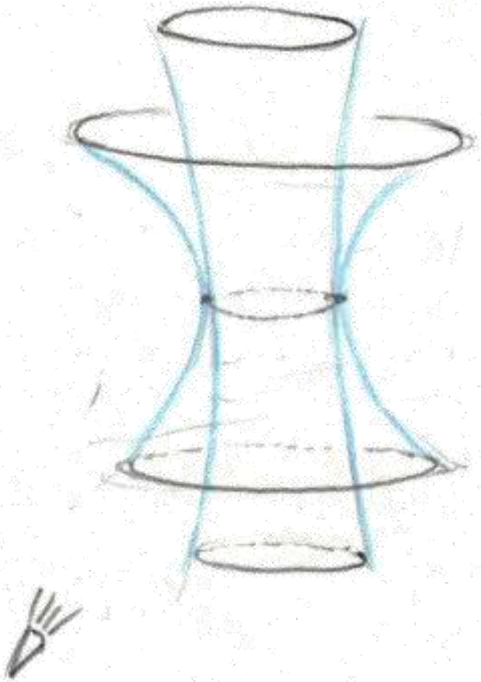
Räumlicher Beweis von Penrose (1951)



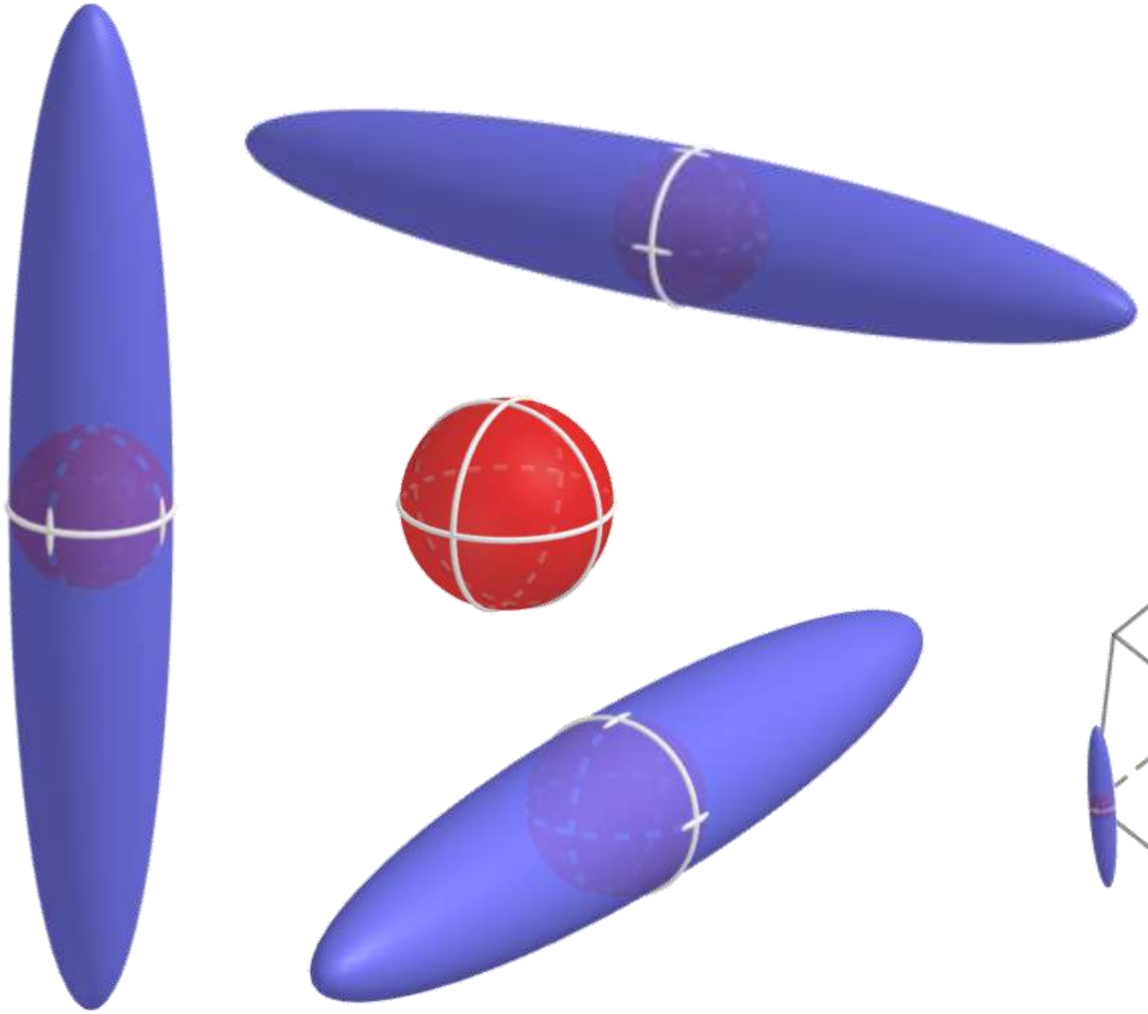


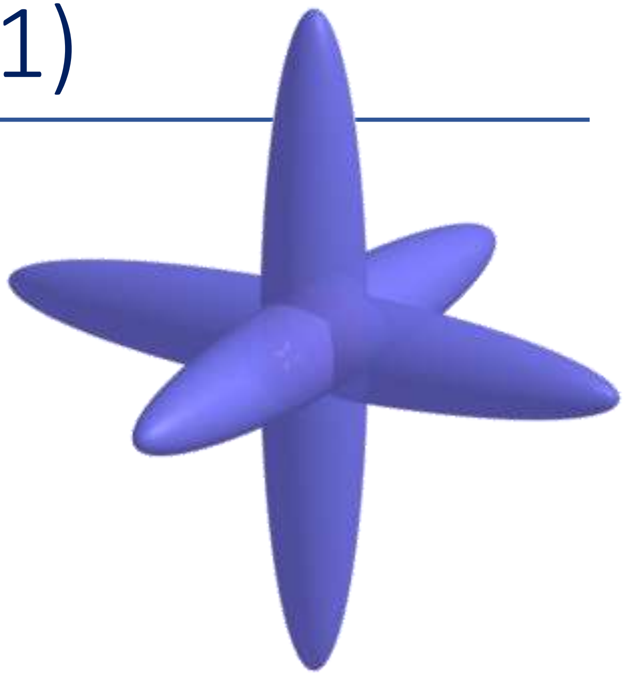
Schnitt oder Schein von ***Regelflächen in Ringkontakt:***

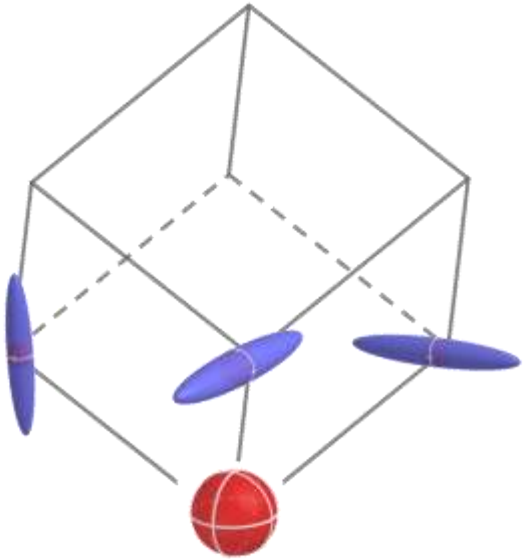


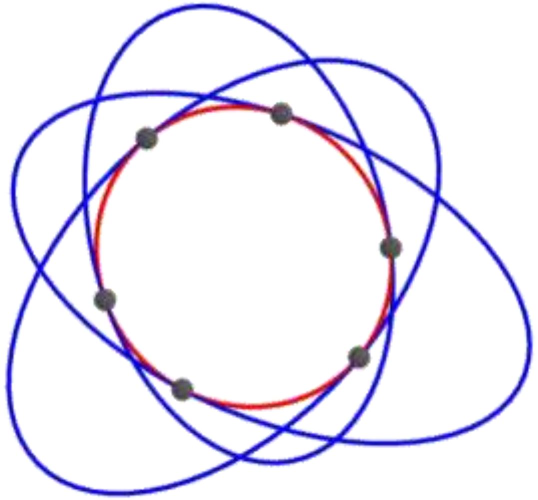


Räumlicher Beweis von Penrose (1951)

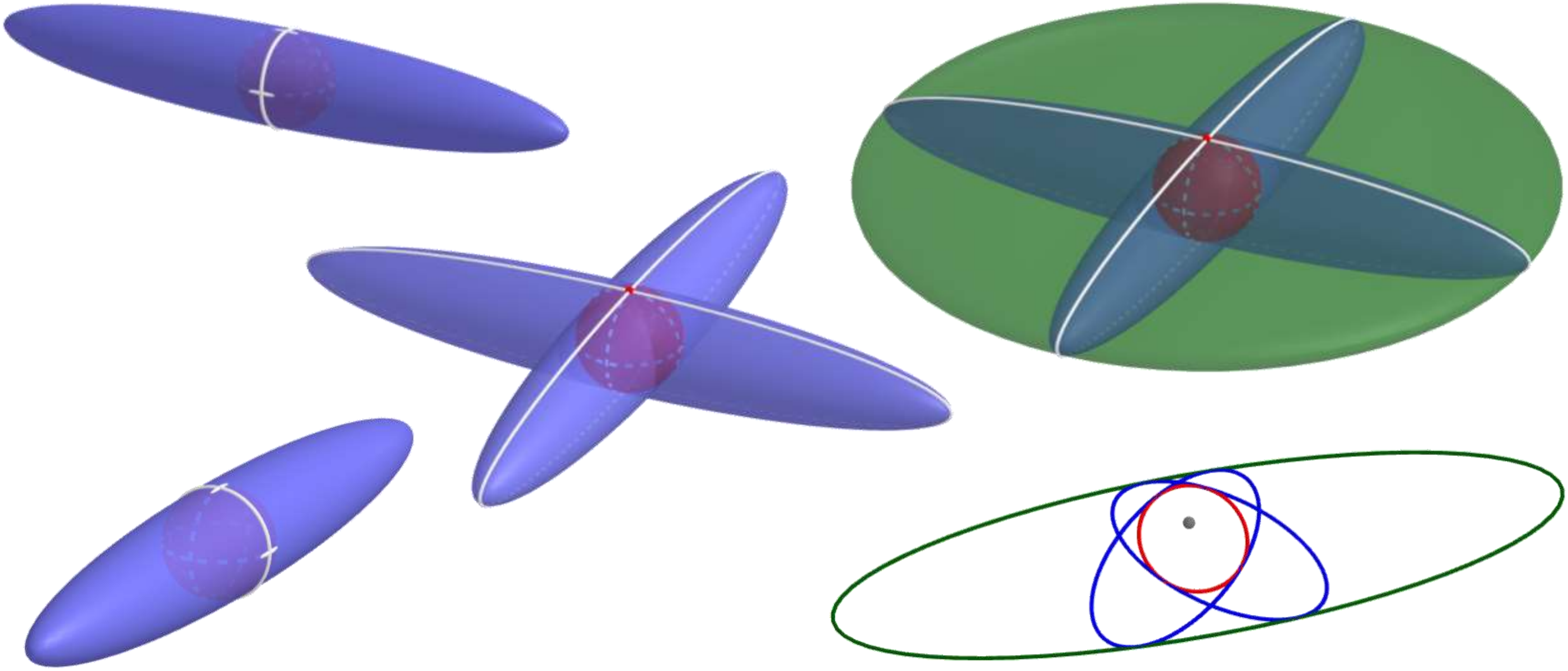




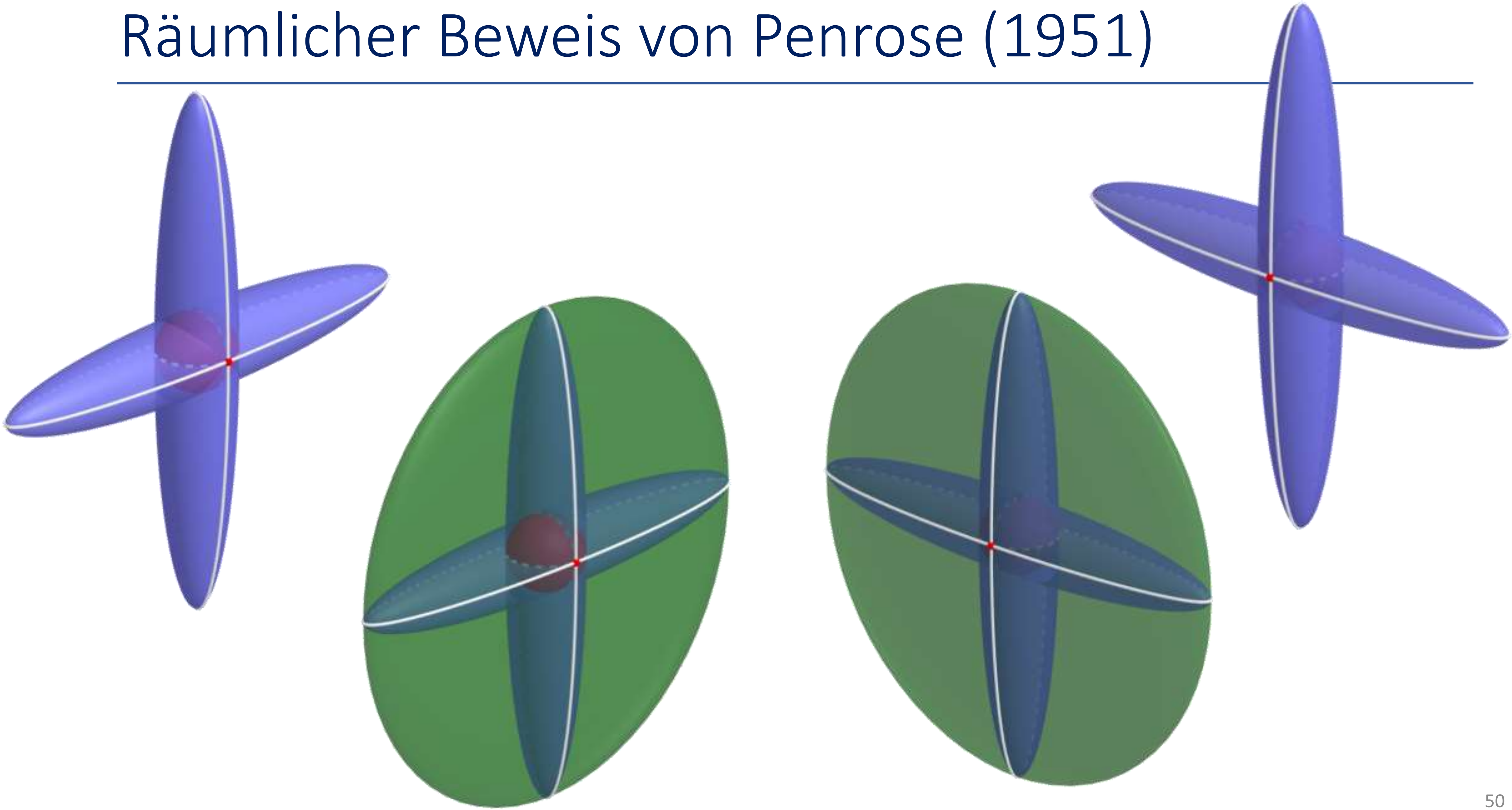




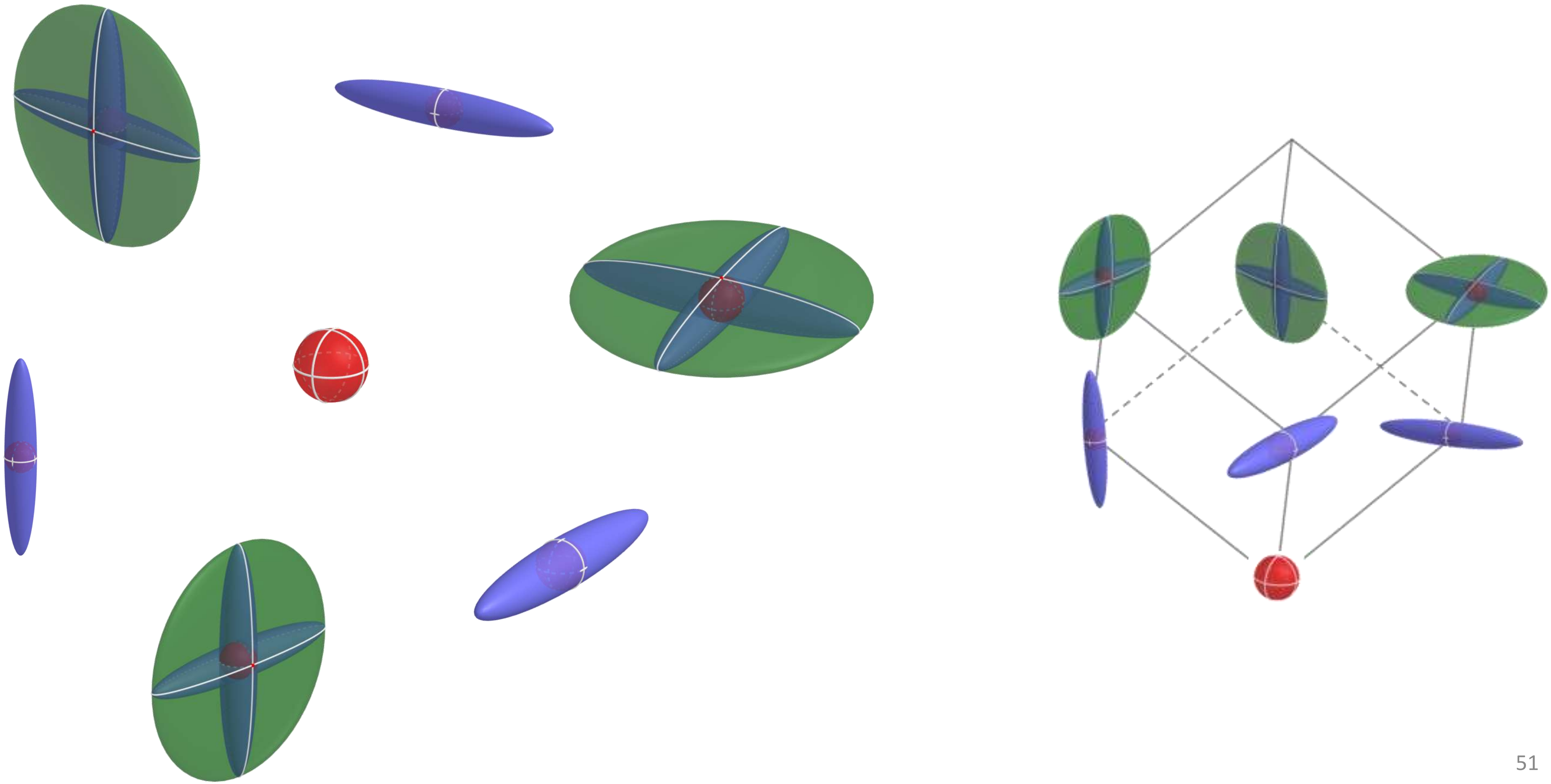
Räumlicher Beweis von Penrose (1951)



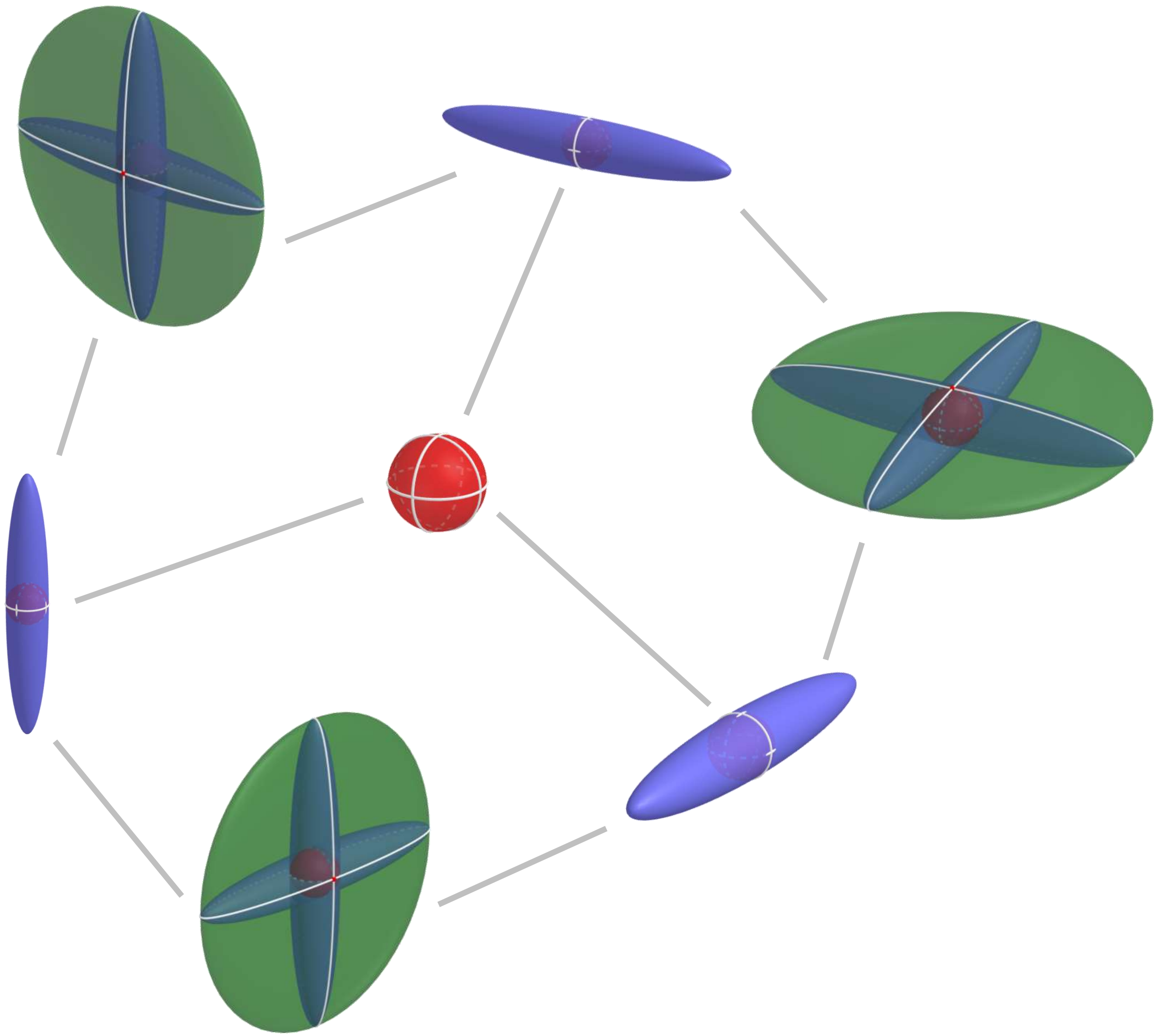
Räumlicher Beweis von Penrose (1951)

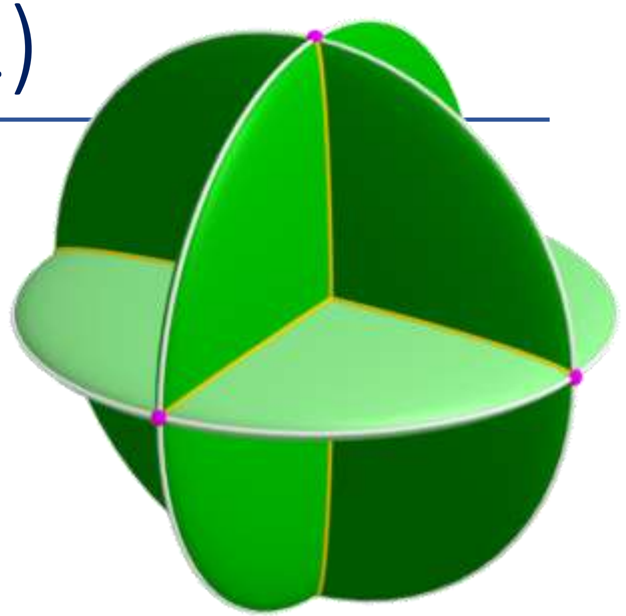


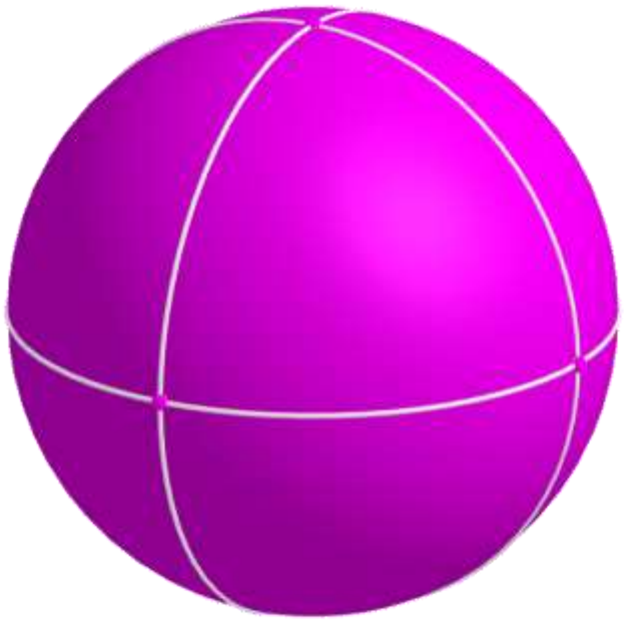
Räumlicher Beweis von Penrose (1951)



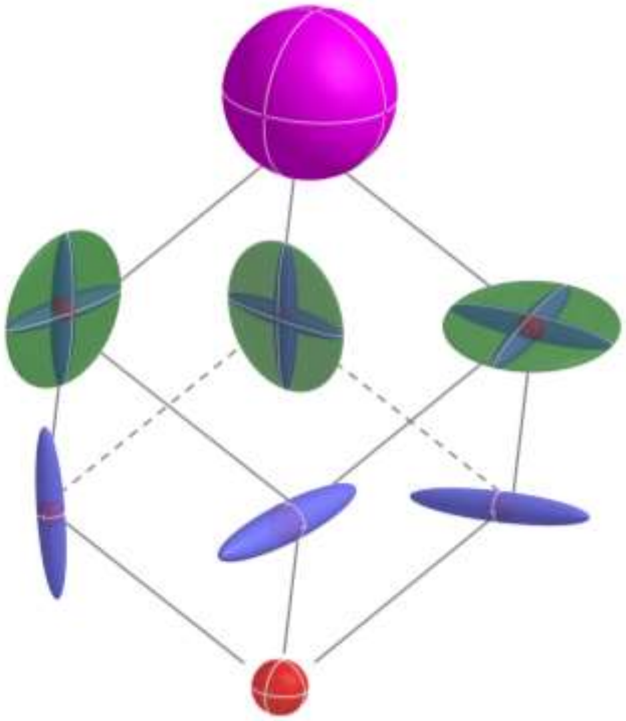
Räumlicher Beweis von Penrose (1951)

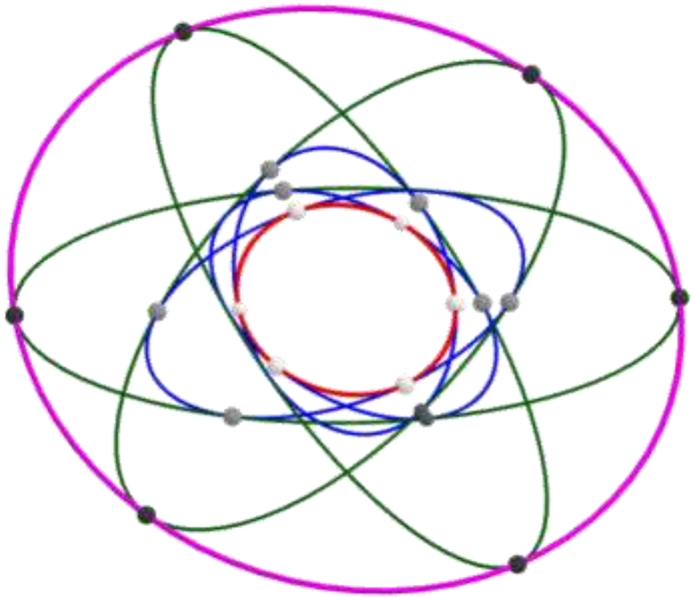






Räumlicher Satz von Penrose (1951)





Penrose: Es seien 7 reguläre Quadriken entsprechend den Ecken eines Würfels gegeben, so dass Quadriken benachbarter Ecken Ringkontakt miteinander haben (**). Dann gibt es eine 8. Quadrik ebenfalls mit dieser Eigenschaft.

Jede Penrose-Konfiguration aus Kegelschnitten ergibt sich als ebener Schnitt einer Penrose-Konfiguration aus Quadriken.

Was macht die räumliche Konfiguration einfacher?

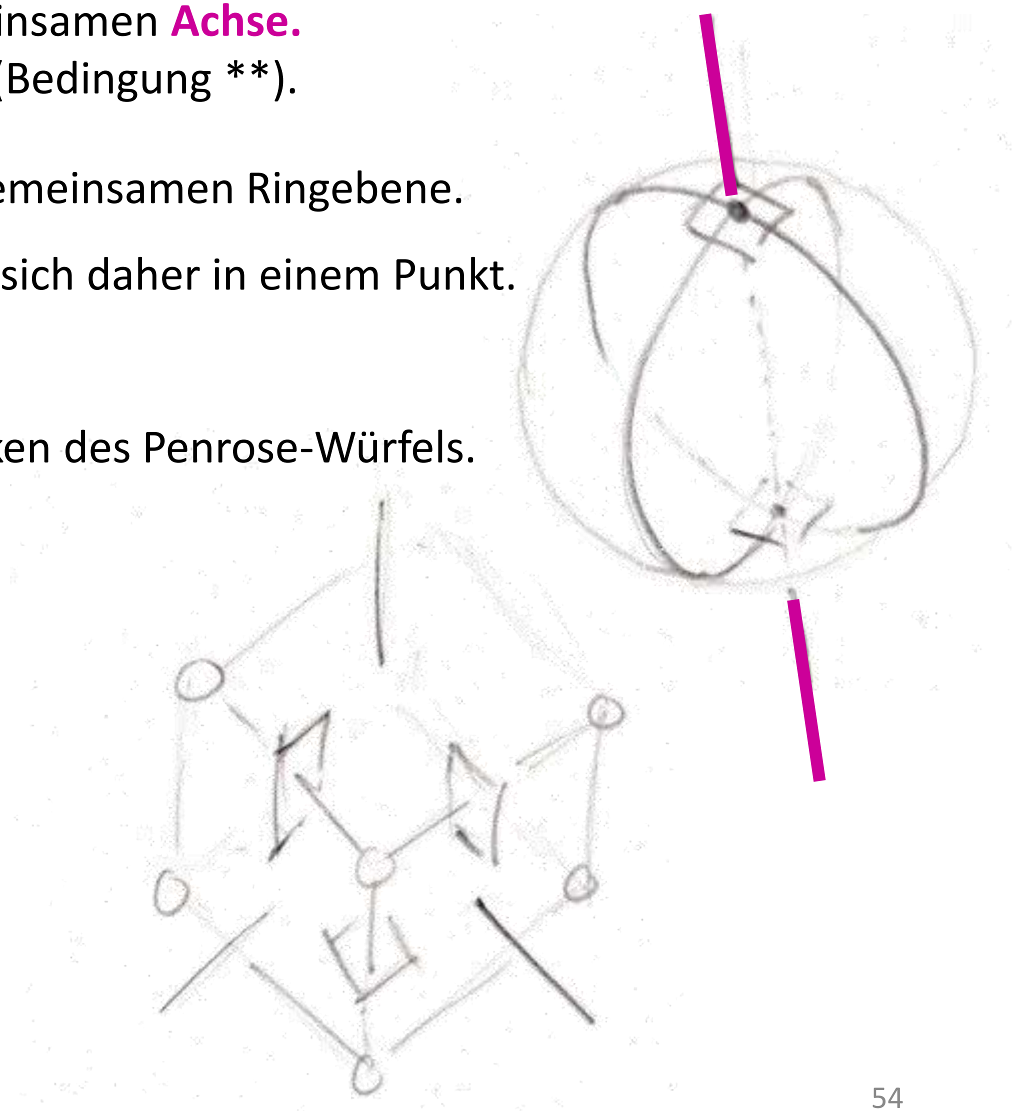
- Die Ringebenen in einer Würfelfläche treffen sich in einer gemeinsamen **Achse.**
  Das ist abgesehen von seltenen Spezialfälle automatisch erfüllt (Bedingung **).

- Die 2 Achsen von benachbarten Würfelflächen liegen in einer gemeinsamen Ringebene.

  Die 3 Achsen von Würfelflächen um eine Würfelecke schneiden sich daher in einem Punkt.

  → Alle Achsen des Würfels schneiden sich in einem **Punkt O.**

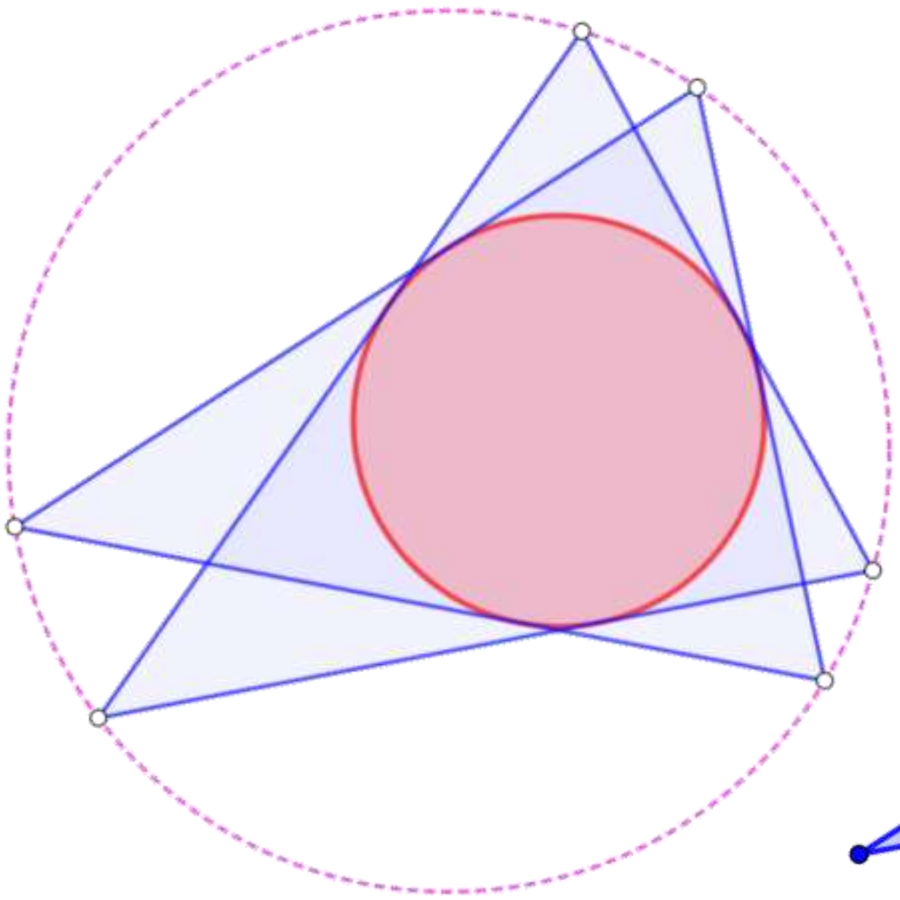
  → Dieser Punkt ist polar zu einer **Ebene ω** bzgl. aller 8 Quadriken des Penrose-Würfels.

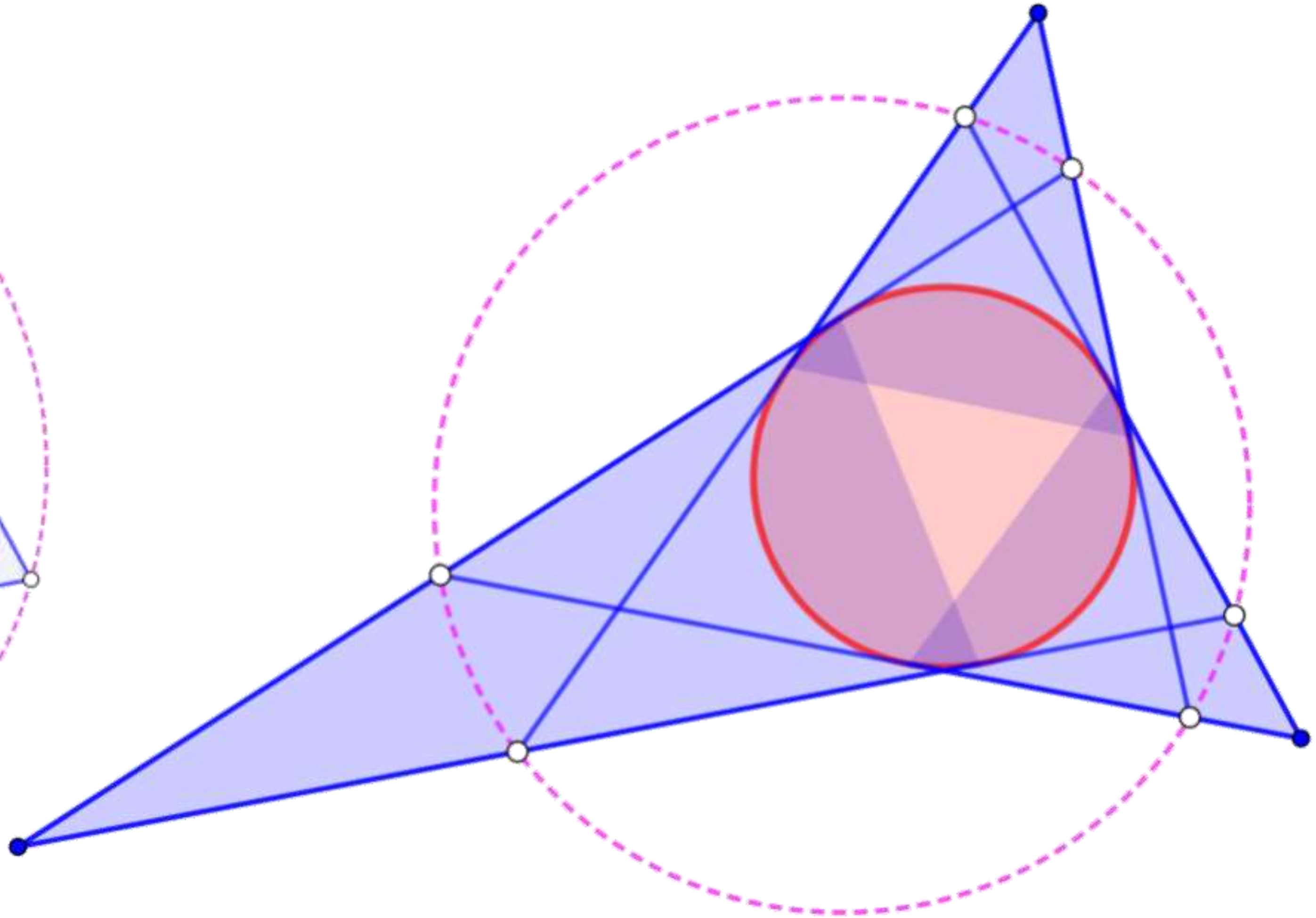


- Bei einem ebenen Schnitt der räumlichen Konfiguration geht der Punkt O verloren.

- In der Ebene haben nur die 4 Kegelschnitte einer Würfelfläche ein gemeinsames Pol-Polaren-Paar.

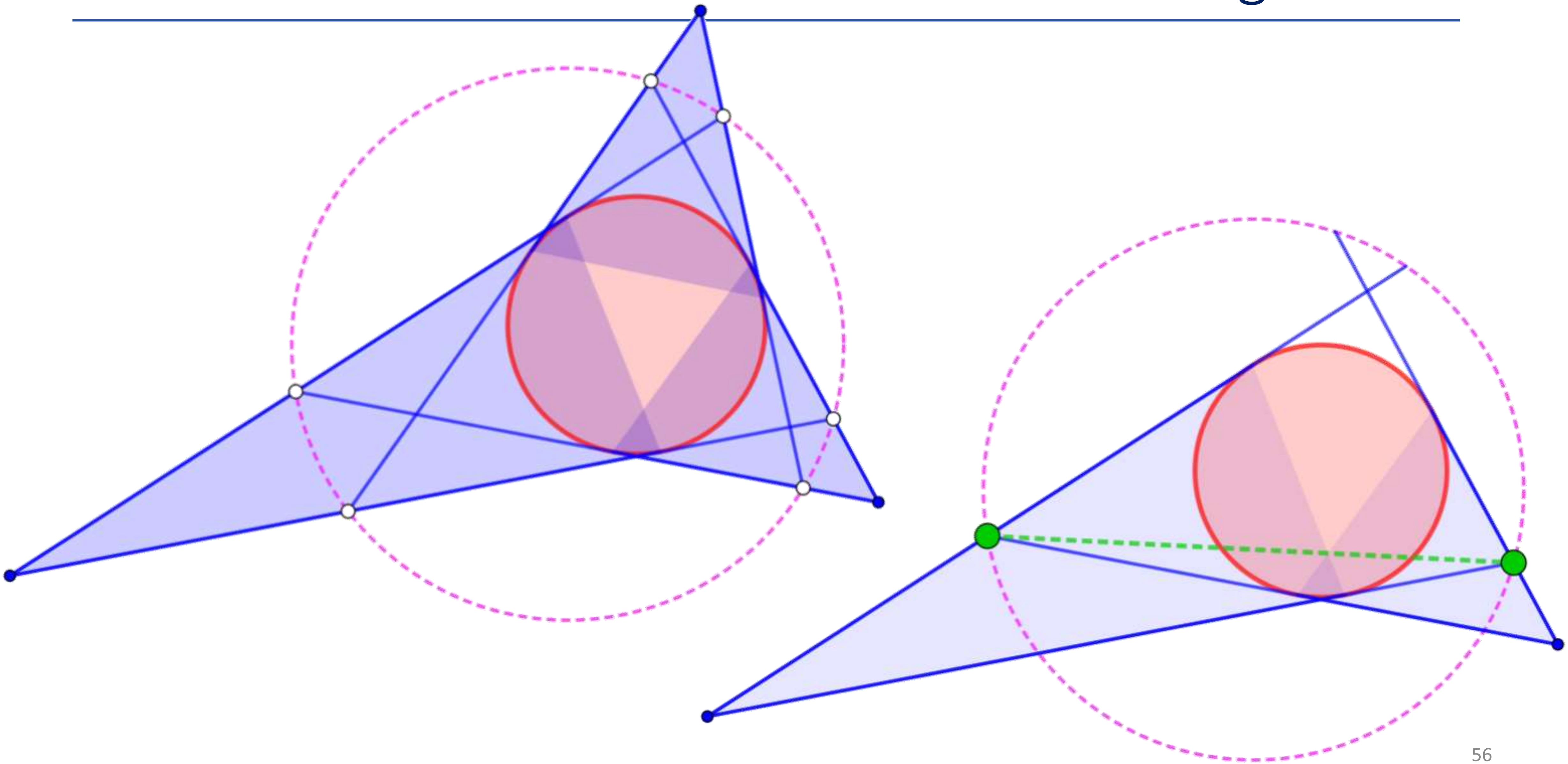
Poncelet als Schnitt einer räumlichen Konfiguration



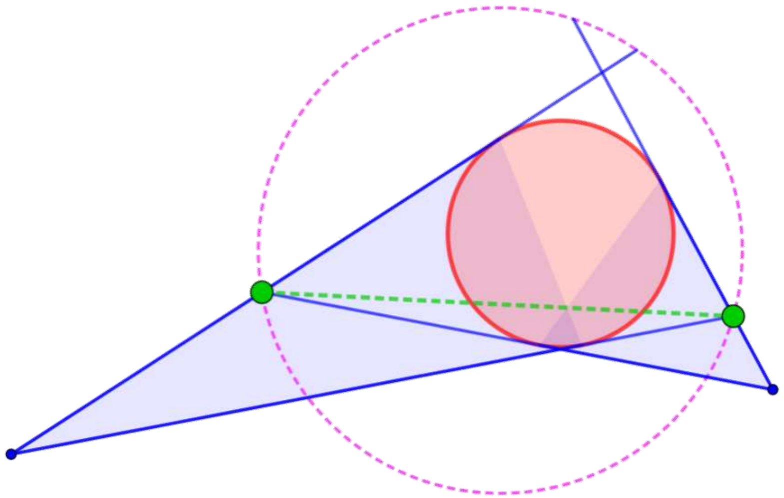


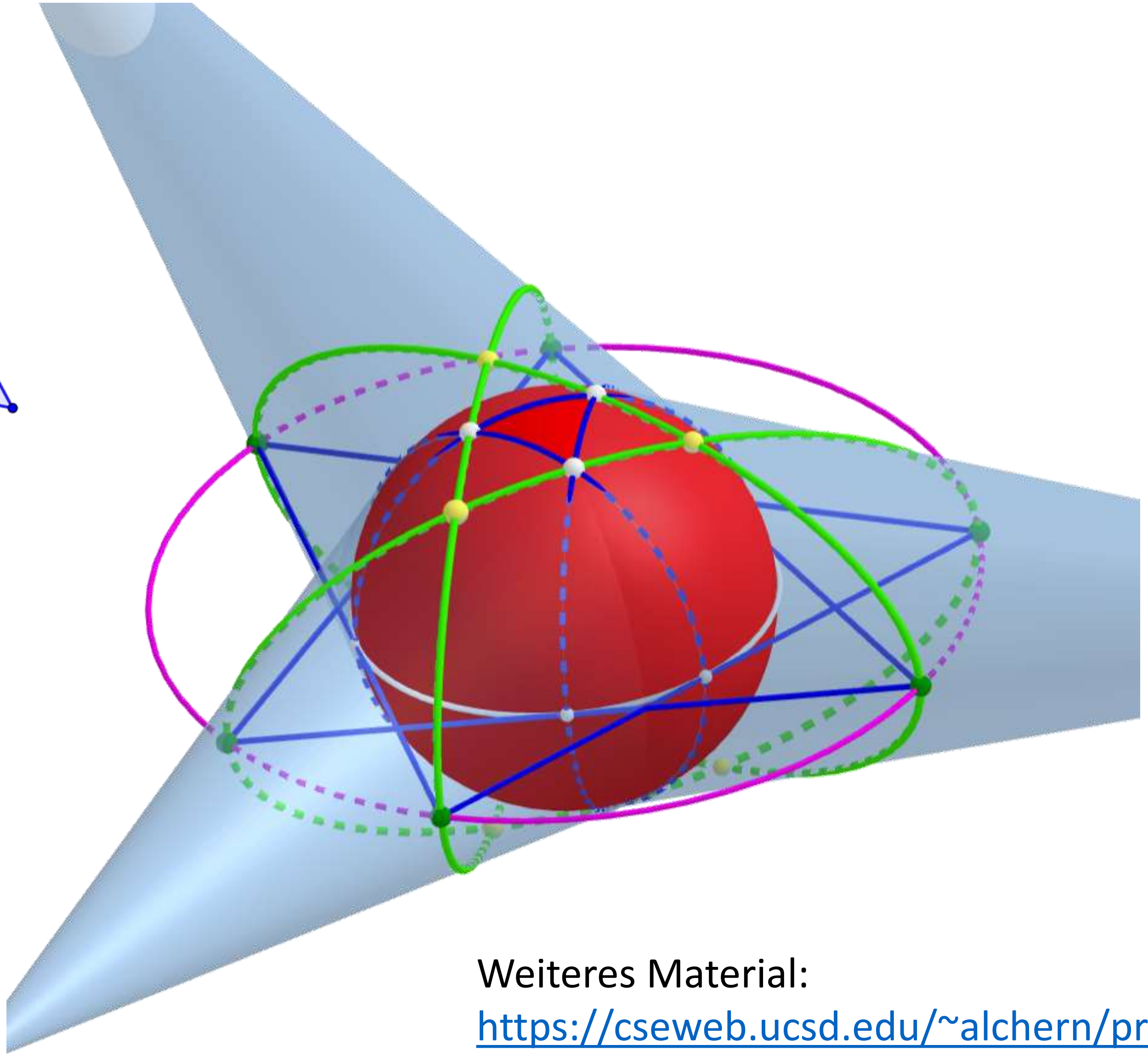
Warum existiert der äußere Kreis?

Poncelet als Schnitt einer räumlichen Konfiguration



Poncelet als Schnitt einer räumlichen Konfiguration





Weiteres Material:
https://cseweb.ucsd.edu/~alchern/projects/Penrose/

Veröffentlichung (preprint): https://arxiv.org/abs/2409.17150

Penrose's eight-conic theorem via Penrose's eight-quadric theorem

Russell Arnold* Albert Chern† Morten Eide‡ Charles Gunn§
Thomas Neukirchner¶ Roger Penrose‖

January 17, 2025

Abstract

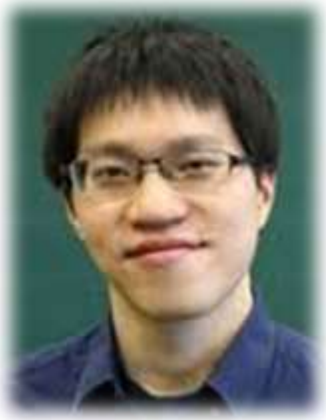
This article proves the following theorem, first enunciated by Roger Penrose about 70 years ago: In $\mathbb{R}P^2$, if regular conics are assigned to seven of the vertices of a combinatorial cube such that (i) conics connected by an edge are in double contact, and (ii) the chords of contact associated to a cube face meet in a common point, then there exists an eighth conic such that the completed cube satisfies (i) and (ii). This conic is unique if not all the common points of condition (ii) are the same. The proof is based on the following analogous theorem, which is also proved: In $\mathbb{R}P^3$, if regular quadrics are assigned to seven of the vertices of a combinatorial cube such that (i) quadrics connected by an edge are in ring contact, and (ii) the ring planes associated to a cube face meet in a common axis, then there exists an eighth quadric such that the completed cube satisfies (i) and (ii). This quadric is unique if not all the common axes of condition (ii) are the same.

Unsere Arbeitsgruppe

Charles Gunn
(Berlin)

Roger
Penrose
(London)

Morten
Eide
(Oslo)



Albert Chern
(San Diego)

Russell
Arnold
(Dornach)

Thomas
Neukirchner
(Karlsruhe)